Math 314: Linear Algebra - Fall 06
Quiz 2: Matrices and linear dependence　　　　Name ______________________

1. The matrix B is the echelon form of the augumented matrix $[A|\mathbf{b}]$, which corresponds to a certain system of linear equations:

$$B = \begin{pmatrix} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Decide if the following statements are true or false, and give a brief justification for your answer.

(a) B is the reduced echelon form of $[A|\mathbf{b}]$.

(b) At least one row of the system $[A|\mathbf{b}]$ is a linear combination of the others.

(c) The system corresponding to the matrix $[A|\mathbf{b}]$ has just one solution.

(d) $\mathbf{b}$ is a linear combination of the colums of A.

(e) $\text{rank}([A|\mathbf{b}]) = 2$.

2. Prove that $\mathbf{v} = [1, 2, 0], \mathbf{w} = [0, 0, 2], \mathbf{x} = [0, 2, 1]$ are linearly independent. Prove that R^3 is the $\text{span}(\mathbf{v}, \mathbf{w}, \mathbf{x})$.

3. Prove that if $\mathbf{x}, \mathbf{y}, \mathbf{z}$ are linearly independent then $\mathbf{x} - \mathbf{y}, \mathbf{x} - \mathbf{z}, \mathbf{y} + \mathbf{z}$ are linearly independent.