Math 314: Homework 1
Fall 2006

Group name _____

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1. Prove that given three vectors $\mathbf{v}, \mathbf{w}, \mathbf{z} \in R^n$ one has $\mathbf{v} \cdot (\mathbf{w} + \mathbf{z}) = \mathbf{v} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{z}$.

2. Let $\mathbf{v}, \mathbf{w} \in R^n$, prove that $\mathbf{v} - \text{proj}_{\mathbf{w}} \mathbf{v}$ is orthogonal to $\mathbf{w}$.

3. (a) Write the normal equation of the lines $\mathcal{S}$ and $\mathcal{P}$ passing through $P(9, 0, 1)$ and $Q(1, 0, 0)$ and orthogonal to the vector $\mathbf{n} = [1, 2, 3]$.

   (b) Find the distance between $\mathcal{S}$ and $\mathcal{P}$.

   (c) Find a line $\mathcal{R}$ parallel to $\mathcal{P}$ such that the distance between the two lines is 4 units.

4. Let $\mathbf{v} = [3, 2, 1]$ and $\mathbf{w} = [0, 1, 0]$ be two vectors in R^3. Write the vector $[15, 9, 5]$ as linear combination of $\mathbf{v}$ and $\mathbf{w}$.

5. Let $\mathbf{w}$ be a vector in R^n which is perpendicular to $\mathbf{v}_1$ and $\mathbf{v}_2$. Prove that $\mathbf{w}$ is perpendicular to any linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.