

Review Problems for Exam 2

This is a list of problems to help you review the material which will be covered in the second exam. Please go over the problem carefully. Keep in mind that I am going to put some problems that are part of what was covered in the first exam. It is a good idea to re-work the problems from the review sheet for the first exam. Enjoy, and have a wonderful fall break.

1. (a) Show that the two functions

$$f(x) = 2 \sin^2 5x \quad g(x) = 3 \cos^2 5x$$

are linearly independent on the real line.

- (b) Find a non-trivial linear combination of the functions

$$f(x) = 2 \sin^2 5x \quad g(x) = 3 \cos^2 5x \quad h(x) = 6$$

that vanishes identically on the real line. Does this contradict part (a)?

2. Solve the initial value problem

$$y'' - 2y' + 2y = 2x; \quad y(0) = 1, \quad y'(0) = 5.$$

3. Consider the differential equation

$$y^{(3)} + y'' = 0.$$

- (a) Find the general solution.

- (b) Show that all solutions with $y(0) = 2$ have the form

$$A + Bx + (2 - A)e^{-x},$$

where A and B are real constants.

- (c) Find a solution with $y(0) = 2$, $y'(0) = 0$ and $y''(0) = 0$.

4. Consider the non-homogeneous linear differential equation

$$3y' + 2y = 2. \tag{**}$$

- (a) Write down the corresponding homogeneous equation.

- (b) Write down the characteristic equation to and solve it.

- (c) Find the general solution the homogeneous equation.

- (d) Find by inspection a particular solution to (**).

- (e) Write down the general solution to (**).

5. Find the general solution of the equation $9y^{(3)} + 11y'' + 4y' - 14y = 0$ knowing that $y = e^{-x} \sin x$ is a solution.

6. Find a particular solution of the differential equation $4y'' + 4y' + y = 3xe^x$. Then write the general solution of such equation.

7. Find a particular solution of the differential equation $y'' + 3y' + 2y = 4e^x$. Then write the general solution of such equation.
8. Write all the solutions of the equation $x^{10} + 1 = 0$.
9. Show that the functions $f(x) = x^2$, $g(x) = x$ and $h(x) = 3x + 5x^2$ are linearly dependent.
10. Consider the differential equation $x^2y'' + xy' - 9y = 0$ with $y > 0$. Check that $y_1(x) = x^3$ is a solution of such equation. Then, use the method of reduction of order to find a second, linearly independent solution.
11. Consider the system of first order differential equations associated to $x^{(4)} + 3x^{(3)} - x = \sin t$. Do not solve the system.
12. Solve the system of linear differential equations $x' = 3y$, $y' = 3x + y$.