REVIEW SHEET

Even though this practice set addresses most of the important topics covered so far, it is still your responsibility to go over the homeworks and notes in order to refresh all the material to date. **The exam always may (will) contain other problems that are non-standard, however, based on the material discussed in class and assigned in homeworks.**

(1) Find what functions of the form $y = \cos(Ax)$ satisfy the (fourth-order) equation
$$y^{(iv)} = 21y$$
(2) Identify whether the initial value problem is separable and/or first-order linear. Solve it
   (a) $x + \frac{dy}{dx} = xy^2$
   (b) $\frac{\sin x}{y'} = \frac{\cos y}{\cos x}$
   (c) $y' + y = \sqrt{(1 - e^t)^5}$
   (d) $(\tan x)y = \sec^2 x - y'$
   (e) $(x + 2y)y' = x + y$.
(3) Show that the following equation is exact and solve it (hint: first bring it to the standard form)
$$\left(\frac{x^2}{4\sqrt{y}}\right)\frac{dy}{dx} = -x\sqrt{y}$$
(4) Solve the homogeneous differential equation:
$$2\frac{dy}{dx}x^2 = (x + y)^2$$
(5) Find a general solution of this (reducible) second-order equation. If helpful assume that x, y and/or y' are positive:
$$y^3y'' = 1$$
(6) What are the consequences of the existence/uniqueness theorem discussed in class regarding the solution to:
$$y' = \frac{e^{xy}}{x - y}, \qquad y(1) = -1$$
(7) Use Euler method with step-size 0.5 to estimate $y(1)$ for the equation:
$$y' = 2xy^2, \qquad y(0) = 1$$

(8) A tank contains 2 gal of ethanol and 1 gal of water. Pure water is pumped into it at the rate 1 gal/min, the contents are mixed and then pumped at the same rate into another tank which initially contains 2 gal of pure water. The drain in the second tank is sealed. What is the concentration of ethanol in the second tank after 10 min?

(9) Consider the population of lady-bugs in Lincoln: The births occur at the rate which is equal to the number of the ladybugs, and the rate of deaths is constant and equal to 1000.
- Set up the differential equation describing the population model.
- What are the critical solutions of such an equation?
- Study the stability of such solutions.
- Interpret those in term of the lady-bugs population.

(10) With the phase diagram, study the stability of the differential equation $y' = (2-x)^3$. Next, solve explicitly the differential equation, and then use some particular solutions to visualize the stability(or non-stability) of the critical solutions.

(11) A paratrooper jumps from an altitude 10000 ft, freely falls for 30 sec, and opens his parachute which provides him with a drag 1.3 (assume the *resistance is proportional to the velocity*). How long will it take him to reach the ground? What is his terminal speed (after opening the parachute)? (*Consider the air drag* $\rho = 0.15$)

(12) After 200 years a certain radioactive substance has decayed to 5% of its initial amount. Find the half-life.

(13) Right after a shutdown the temperature of a reactor is 1000° C and (at that instant) is cooling down at the rate 20 deg per second. The cooling system is comprised of pipes running water at 10 C through the reactor. What will be the temperature of the reactor after 2 min?