QUIZ 1–SOLUTIONS

(1) Find an explicit solution to the initial value problem

$$\frac{dy}{dx} - 2x + 1 = 0; \quad y(0) = 2.$$

Solution *Isolate the derivative and integrate.*

$$\begin{aligned} \frac{dy}{dx} &= 2x - 1 \\ y &= \int (2x-1)\, dx \\ &= x^2 - x + C \end{aligned}$$

Use the initial condition to solve for C.

$$\begin{aligned} 2 &= y(0) \\ &= 0^2 - 0 + C \\ &= C \end{aligned}$$

The solution is $\boxed{y = x^2 - x + 2.}$

(2) Determine whether the existence and uniqueness theorem (Theorem 1 in section 1.3) guarantees existence of a solution to the initial value problem

$$\frac{dy}{dx} = \sqrt[3]{y}; \quad y(0) = 0.$$

Find an explicit solution to this initial value problem.

Solution *The function*

$$f(x, y) = \sqrt[3]{y}$$

is continuous in the entire real plane, but the y-derivative

$$D_y f(x, y) = \frac{1}{3}(y)^{-2/3}$$

is not continuous at $(0, 0)$.

$\boxed{\textit{Theorem 1 from section 1.3 does not guarantee existence of a solution with } y(0) = 0.}$

A solution to this initial value problem is $\boxed{\textit{the constant function } y = 0.}$

(3) Determine whether the existence and uniqueness theorem (Theorem 1 in section 1.3) guarantees existence of a solution to the initial value problem

$$\frac{dy}{dx} = \sqrt[3]{y - 2}; \quad y(0) = 2.$$

Date: September 5, 2007.

Find an explicit solution to this initial value problem.

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$$D_y f(x, y) = \frac{1}{3}(y - 2)^{-2/3}$$

is not continuous at $(0, 2)$.

$\boxed{\textit{Theorem 1 from section 1.3 does not guarantee existence of a solution with } y(0) = 2.}$

A solution to this initial value problem is $\boxed{\textit{the constant function } y = 2.}$