Final

This is a closed book exam.You have two hours. Solve the following exercises and write clear solutions to receive partial credit. Good Luck.

1. (a) Find the general solution of the differential equation:

$$y'' + y = 0$$

(b) Solve the initial value problem

$$\begin{cases} y'' + y = 6\sin x \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

2. Find the general solution of the differential equation $y^{''} + 2y' + y = 0$.

3. Consider the differential equation $\frac{dy}{dx} = \sqrt{x - y}$. Discuss the existence and uniqueness of this differential equation in terms of the theorem that we discussed in class, with the initial condition $y(2) = 2$.

4. Find the general solution to the differential equation

$$xe^y dy + (e^y + x^3 e^{2x})dx = 0$$

5. Suppose the population $P(t)$ of a country satisfies the differential equation

$$\frac{dP}{dt} = kP(200 - P)$$

with k a constant (with P given in millions). Its population in 1940 was 100 million and was then growing at a rate of 1 million per year. Predict this country's population in the year 2010.

6. Find the solution to the following initial value problem using Laplace transforms.

$$\begin{cases} x'' - x' - 2x = 1 \\ x(0) = 5 \\ x'(0) = 0 \end{cases}$$

7. Consider the sistem of differential equations:

$$\begin{cases} \frac{dx}{dt} = 3x + y \\ \frac{dy}{dt} = x + y. \end{cases}$$

(a) Write the system in matrix form.

(b) Find the eigenvalues.

(c) Give the general solution of the system.

(d) Solve the initial condition problem with $x(0) = 0$ and $y(0) = 1$.