Midterm 3
Mathematics 221
Fall 2005 Name ____________________
You must justify your answer to receive credit for a given solution.

1. Consider the following system of differential equations

$$(*) \qquad \frac{dx_1}{dt} = x_1 - \frac{2}{3}x_2 \quad \text{and} \quad \frac{dx_2}{dt} = 3x_1 - 1x_2$$

   (a) Write the system $(*)$ in matrix notation; that is, on the form $\mathbf{x}' = \mathbf{A}\mathbf{x}$.
   (b) Find the solution general solution to $(*)$.

2. Find the determinant of the following matrix.

$$\mathbf{A} = \begin{bmatrix} 1 & 5 & -6 & 0 \\ 1 & 3 & -4 & 0 \\ 1 & -3 & 6 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

3. Consider the system of differential equations

$$(*) \qquad \frac{dx_1}{dt} = -2x_1 + x_2 \quad \text{and} \quad \frac{dx_2}{dt} = -x_1 - 4x_2$$

(a) Write the system $(*)$ in matrix notation; that is, on the form $\mathbf{x}' = \mathbf{B}\mathbf{x}$.

(b) Find the eigenvalues of $\mathbf{B}$.

(c) Find the general solution of the system $(*)$.

(c) Find the particular solution of the system $(*)$ corresponding to the initial value $x_1(0) = 0$ and $x_2(0) = 1$.

4. Consider the system of differential equations

$$(*) \qquad \frac{dx}{dt} = x \quad \text{and} \quad \frac{dy}{dt} = 3y$$

(a) Find the critical points.

(b) Check that $x(t) = x_0 e^t$, $y(t) = y_0 e^{3t}$ is a solution of the system.

(c) Find the equation of the trajectories.

(d) Show that the critical point is a node.

(e) Show that the critical point is not stable.

5. The homogeneous equation

(∗)
$$y'' + 3y' + 2y = 0$$

has the general solution
$$y_c = c_1 e^{-x} + c_2 e^{-2x},$$

where c_1 and c_2 are real constants.

(a) Find a particular solution to the nonhomogeneous equation

(1)
$$y'' + 3y' + 2y = x$$

(b) Find a particular solution to the nonhomogeneous equation

(2)
$$y'' + 3y' + 2y = e^{-x}$$

(c) Find the general solution to the nonhomogeneous equation

(∗∗)
$$y'' + 3y' + 2y = x + e^{-x}$$

(d) Find the solution to (∗∗) with $y(0) = 0$ and $y'(0) = 0$.