

Exam 2

This is a closed book exam. Solve the following exercises and write clear solutions.

1. (a) Consider the following homogeneous differential equation

$$y^{(3)} - 6y^{(2)} + 9y' = 0.$$

Find the general solution.

- (b) Consider the following differential equation

$$y^{(3)} - 6y^{(2)} + 9y' = e^x.$$

Find the general solution.

2. Consider the following differential equation

$$y^{(2)} - 6y' + 9y = e^{3x}.$$

Find the general solution.

3. (a) Show that the real functions $f(x) = 3x^2$ and $g(x) = x^3$ are linearly independent.

(b) Show that the real functions $f(x) = 3x^2$, $g(x) = x^3$, and $h(x) = 6x^2 - 5x^3$ are linearly dependent.

4. Consider the differential equation

$$y^{(3)} + y'' = 0.$$

(a) Find the general solution.

(b) Show that all solutions with $y(0) = 2$ have the form

$$A + Bx + (2 - A)e^{-x},$$

where A and B are real constants.

(c) Find a solution with $y(0) = 2$, $y'(0) = 0$ and $y''(0) = 0$.

5. Find the solution of the system

$$\begin{cases} x' = y \\ y' = 2x + y \end{cases}$$

6. Use the separation of variables to find the general solution for $yy' = x(y^2 + 1)$.

7. Find all solutions of the equation $x^6 - 1 = 0$