

Exam 1
Math 221
Spring 2005

Name _____

Instructions: Make sure your exam has all six pages. You must justify your answer to receive credit for a given solution.

1. (15 points) Determine whether the existence and uniqueness theorem (Theorem 1 in section 1.3) guarantees existence of a solution to the initial value problem

$$\frac{dy}{dx} = \sqrt[3]{y-2}; \quad y(0) = 2.$$

Find an explicit solution to this initial value problem.

2. (10 points) A 400-gal tank initially contains 100 gal of brine containing 50 lb of salt. Brine with a concentration of 1 lb salt per gallon enters the tank at the rate of 5 gal/s, and brine flows out of the tank at the rate of 3 gal/s.

Set up the appropriate differential equation.

Do not solve the differential equation but describe how you would use the solution to determine how much salt the tank will contain when it is full of brine.

3. (15 points) Suppose that a population of a certain species consists initially of 100 elements. Assume that the death rate $\delta = 0$, and the birth rate $\beta = 0.0005P(t)$. Has this species a problem of extinction?

4. (15 points) Find the critical points of the differential equation

$$\frac{dx}{dt} = 3x - x^2.$$

Then analyze the sign of the right-hand side of the differential equation to determine whether each critical point is stable or unstable, and construct the corresponding phase diagram for the differential equation.

5. (10 points) Find the solution of the initial value problem

$$x^3 y' + 2x^2 y = 1, \quad y(1) = 1$$

6. (10 points) Find the general solution of $(x^3 + \frac{y}{x})\frac{dx}{dy} + (y^2 + \ln(x)) = 0$,