

Name \_\_\_\_\_

**Review**

This is a list of integrals you should know how to compute in order to pass the 3th test. For each integral, draw the region or the volume of integration, set up clearly the integral, say which set of coordinates you are using. Have fun.

1. Find the volume of the solid bounded by  $x = y^2 + z^2$ ,  $x = 0$  and  $y^2 + z^2 = 9$ .

$$\int_0^3 \int_0^{2\pi} \int_0^{\sqrt{9-x}} 1 r dz d\phi dr.$$

2. Find the volume bounded by  $z = x^2 + y^2 + 4$  and the planes  $x = 0$ ,  $y = 0$ ,  $z = 0$  and  $x + y = 1$ .

$$\int_0^1 \int_0^{1-x} (x^2 + y^2 + 4) dy dx.$$

3. Find the volume of the solid bounded below by  $z = 4 - x^2$  and above by the plane  $z = 4 - x$ , and by the planes  $y = 5$  and  $y = 0$ .

$$\int_0^5 \int_0^1 \int_{4-x}^{4-x^2} 1 dz dx dy.$$

4.  $\int_0^1 \int_{x^2}^1 x^3 \sin(y^3) dy dx$ .

5. Find the volume inside  $x^2 + y^2 + z^2 = 16$  and outside the half-cone  $z = \sqrt{x^2 + y^2}$ .

$$\int_0^4 \int_{\pi/4}^{\pi} \int_0^{2\pi} r^2 \sin(\phi) d\theta d\phi dr.$$

6.  $\iiint_Q y dV$ , where  $Q$  is the solid bounded by  $x^2 + z^2 = 16$ ,  $x^2 + z^2 = 9$  and the two planes  $y = 8$  and  $y = 0$ .

$$\int_0^{2\pi} \int_3^4 \int_0^8 y r dy dr d\theta.$$

7.  $\iiint_Q yz dV$ , where  $Q$  is the solid bounded by  $z = 0$ ,  $z = y$  and  $x^2 + y^2 = 9$ , with positive  $y$ .

$$\int_0^3 \int_0^{\pi} \int_0^{r \sin(\theta)} z r^2 \sin(\theta) dz d\theta dr.$$

8.  $\iint_D x dR$ , where  $D$  is the region inside  $x^2 + y^2 = 4$  and outside  $x^2 + y^2 = 2x$ .

The second curve is a circle and the equation in polar coordinate is given by  $r = 2 \cos(\theta)$ , where to get one loop we need  $0 \leq \theta \leq \pi$ .

$$\int_0^{2\pi} \int_0^2 r^2 \cos(\theta) dr d\theta - \int_0^{\pi} \int_0^{2 \cos(\theta)} r^2 \cos(\theta) dr d\theta.$$

9. Compute the area of the surface  $z = xy$  that lies within the cylinder  $x^2 + y^2 = 1$ .  
The partial derivatives are  $\partial z/\partial x = y$  and  $\partial z/\partial y = x$ . By using the surface area formula:

$$\int_0^{2\pi} \int_0^1 \sqrt{1 + r^2 \cos^2(\theta) + r^2 \sin^2(\theta)} r dr d\theta.$$

10. Compute  $\iint_D \cos(x^2 + y^2) dA$ , where  $D$  is the region in the first quadrant between  $x^2 + y^2 = 4$  and  $x^2 + y^2 = 9$  and the lines  $x = y$  and  $y = 0$ .

$$\int_0^{\pi/4} \int_2^3 \cos(r^2) r dr d\theta.$$