Math 208: Calculus - Fall 06
Test 2: Review   Name ________________________________

This review is meant to help you on what you need to focus for the test. It is not meant to replace the book, the homework assigments, or any other activities we do in class.

1. Consider the surface $z^2 = 3x^2 + 3y^2$.
   (a) Sketch the graph of the surface.
   It is a cone.
   (b) How many points are in the surface corresponding to $x = 3$ and $y = 1$
   There are two points $(3, 1, \sqrt{30})$ and $(3, 1, -\sqrt{30})$.
   (c) The surface correspond to the level surface of a three variable function. Which function?
   $F(x, y, z) = 3x^2 + 3y^2 - z^2$.
   (d) Compute the gradient of the function of part 1c.
   $\nabla(F) =< \partial F/\partial x, \partial F/\partial y, \partial F/\partial z >=< 6x, 6y, -2z >$
   (e) Compute the tangent plane to the surface $z^2 = 3x^2 + 3y^2$ at the point $(1, 1, \sqrt{6})$.
   $\nabla(F(1, 1, \sqrt{6})) =< 6, 6, -2\sqrt{6} >$ is the normal vector to the tangent plane. The equation of the tangent plane is $6(x - 1) + 6(y - 1) - 2\sqrt{6}(z - \sqrt{6}) = 0$.

2. Let $f(x, y) = ln(x^2 + y^2 - 9)$ be a function in two variables.
   (a) Find the domain of $f(x, y)$.
   $x^2 + y^2 - 9 > 0$: it is the region outside the circle of radius 3.
   (b) Find the range of the function $f(x, y)$.
   It is the set of all real numbers.
   (c) Draw the countour map of $z = f(x, y)$, in particular draw the level curve corresponding to the value $z = 0$.
   (d) Find the tangent plane to the graph of the function at the point corresponding to $(3, 1)$.
   The gradient of f is $\nabla(f) =< \frac{2x}{x^2+y^2-9}, \frac{2y}{x^2+y^2-9} >$, therefore $\nabla(f(3, 1)) =< 6, 2 >$. The equation of the tangent plane is $z = 6(x - 3) + 2(y - 1)$.
   (e) Give the equation of the linear approximation of the function at the point corresponding $(3, 1)$.
   $L(x, y) = 6(x - 3) + 2(y - 1)$.
   (f) Use the equation in 2e to approximate the value of the function at $(3.001, 1.001)$.
   $L((3.001, 1.001) = 6(0.001) + 2(0.001) = 8.001$.
   (g) Compute the gradient of the function $f(x, y)$ at the point $(3, 1)$.
   $\nabla(f(3, 1)) =< 6, 2 >$.
   (h) Draw the gradient in the countour map you draw in 2c.
   (i) Compute the tangent line to the level curve of the function that passes through the point $(3, 1)$.
   We need a vector $< a, b >$ which is perpendicular to the gradient $< 6, 2 >$. So $< a, b > \dot{<} 6, 2 >= 0$ implies $6a + 2b = 0$. A vector perpendiculr to $< 6, 2 >$ is $< 1, -3 >$. So the equations for the line are $x = 3 + t, y = 1 - 3t$.

(j) Compute the directional derivative $D_{\mathbf{u}}(f(3,1))$ where $\mathbf{u}$ is the unit vector corresponding to the vector $< 1,2 >$.
We need a unit vector $||\mathbf{u}|| = \sqrt{1+4} = \sqrt{5}$. So $D_{\mathbf{u}}(f(3,1)) = 1/\sqrt{5}(< 6,2 > \dot{<}1,2 >) = 1/\sqrt{5}(6+4)$

(k) In which direction does the maximum rate of change of the function f at the point $(3,1)$ occur?
$\nabla(f(3,1)) =< 6,2 >$.

(l) What is the maximum rate of change for the function $f(x,y)$ at the point $(3,1)$.
$||\nabla(f(3,1))|| = \sqrt{36+4}$.

(m) Describe what you obtain intersecting the graph of the function with the plane $x = 3$.
The graph is the curve in the plane $x = 3$, the two curves are symmetric with respect to the line $y = 0, x = 3$.

3. Write the equation of three different surfaces which intersections with planes parallell to the xy-plane are circles.
$x^2 + y^2 = 9$, $z^2 = x^2 + y^2$, $z = x^2 + y^2$.

4. Let $f(x,y) = ln(x^2y) + \sin(xy + y^2)$ be a function in two variables.

(a) Assume that $z = f(x,y)$, $x = t^2e^{t^2}$ and $y = t$, compute $\partial f(x,y)/\partial t$.

$$\frac{\partial f}{\partial x} = \frac{2xy}{x^2y} + y\cos(xy + y^2),$$

$$\frac{\partial f}{\partial y} = \frac{x^2}{x^2y} + (x+2y)\cos(xy + y^2),$$

$$\frac{dx}{dt} = 2t(e^{t^2}) + 2t * t^2e^{t^2}$$

$$\frac{dy}{dt} = 1$$

So

$$\partial f(x,y)/\partial t = (\frac{2t^2e^{t^2}t}{(t^2e^{t^2})^2t} + t\cos((t^2e^{t^2})t + t^2))(2t(e^{t^2}) + 2t*t^2e^{t^2}) + (\frac{(t^2e^{t^2})^2}{(t^2e^{t^2})^2t} + (t^2e^{t^2} + 2t)\cos((t^2e^{t^2})t + t^2))$$

(b) Assume that $z = f(x,y)$, $x = st^{-8}$ and $y = \tan s^2$, compute $\partial z/\partial t$ and $\partial z/\partial s$.
$\frac{\partial x}{\partial s} = t^{-8}$, $\frac{\partial x}{\partial t} = -8st^{-9}$, $\frac{\partial y}{\partial s} = 2s\frac{1}{\cos(s^2)}$, $\frac{\partial y}{\partial t} = 0$.

5. Number 52 page 977.

6. Let $\mathbf{r}(t) =< t^2, \sin(t), \cos(t) >$ be a vector-valued function. Compute the tangent line to the curve traced out by the endpoints of $\mathbf{r}$ at the point corresponding to $t = 2\pi$.
We did this in class

7. Decide whether the following limits exist

$$\lim_{(x,y)\to(0,0)} \frac{x^2+xy}{x^2-y^2} \quad \text{and} \quad \lim_{(x,y)\to(0,0)} \frac{x^2+xy}{x+y^2}$$

We did this in class.