

This is the second test: there are four pages and a total of 100 points. Read carefully the questions and write your answers neatly. Good Luck.

1. (55 points) In the next page and in the first half of the following page $f(x, y) = \sqrt{x^2 + y^2 - 9}$ is a function in two variables.

(a) (5 points) What is the domain of $f(x, y)$? _____

(b) (5 points) What is the range of $f(x, y)$? _____

(c) (5 points) Draw some level curves (at least three) of the countour map of $f(x, y)$ and label them.

(d) (5 points) Compute the gradient $\nabla f(x, y)$.

(e) (10 points) Compute the tangent plane to the graph of the function at the point in the graph corresponding to $(3, 1)$.

(f) (5 points) Use the linear approximation to approximate the value of the function at the point $(3.008, 1.001)$.

(g) (10 points) Let $\mathbf{u} = \langle 1/\sqrt{2}, 1/\sqrt{2} \rangle$, compute the directional derivative $D_{\mathbf{u}}f(3, 1)$.

(h) (5 points) In which direction the maximum rate of change of the function at the point $(3, 1)$ occurs? _____

(i) (5 points) What is the minimum possible directional derivative at $(3, 1)$?

2. (10 points) Consider the surface S given by the equation $z^2 = x^2 + y^2 - 9$. The surface S is not the graph of a function but it is a level surface of a three variable function. Compute the plane tangent to the surface S at the point $(3, 1, -1)$.

3. (10 points) Decide whether the following limit exists

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + xy^2}{x^2 + y^2}.$$

4. (10 points) The point $(4/3, 4/3)$ is a critical point for the function $f(x, y) = 4xy - x^3 - 2y^2$. Decide whether is a local maximum, a local minimum or a saddle point according to the following information:

$$f_{xx}(4/3, 4/3) = -24/3, f_{yy}(4/3, 4/3) = -4, f_{xy}(4/3, 4/3) = 4.$$

5. (10 points) Let $f(x, y) = ye^x$ be a function in two variables. Assume that $x = \sin(3t)$ and $y = t$, compute df/dt .

6. (5 points) What is the difference between the surface $z = x^2 + y^2$ and the surface $z^2 = x^2 + y^2$?