Math 208: Calculus and Analytic Geometry

This is a set of exercise that will help you prepare for the first in-class test.

Vectors

1. Let $\mathbf{v} =< 3, 2, 1 >$ and $\mathbf{w} =< 3, 0, 1 >$. Find $\mathbf{v} \cdot \mathbf{w}$, $\mathbf{v} \times \mathbf{w}$, $||\mathbf{v}||$, $proj_{\mathbf{v}}\mathbf{w}$, a vector of length 4 in the opposite direction of $\mathbf{v}$.

Planes and lines

1. Find the plane passing through the points $P(0, 0, 1)$, $Q(1, 2, 3)$ and $R(4, 1, 2)$.
2. Let $\pi : 3x - 6y + 8z = 0$ be a plane in R^3. Find a line perpendicular to π passing through the point $P(8, 1, 0)$.
3. Let $\pi_1 : 3x - 6y + 8z = 0$ and $\pi_2 : 4x + 2y = 0$ be two planes. Find the intersection of the two planes.
4. Let $\pi_1 : 3x - 6y + 8z = 0$ and $\pi_2 : 4x + 2y = 0$ be two planes. Find the plane perpendicular to π_1 and π_2 passing through the point $P(8, 1, 0)$.
5. Find the distance between the plane $\pi : 3x - 6y + 8z = 0$ and the line with direction the vector $< 2, 1, 0 >$ and passing through the point $P(8, 1, 0)$.
6. Decide whether the two lines r and s as in number 7 are parallel, skew or intersecting.
7. Find a plane parellel to the lines

$$s : \begin{cases} x = t + 9 \\ y = 7t \\ z = 8t + 8 \end{cases} \quad \text{and} \quad r : \begin{cases} x = 5t + 9 \\ y = 4 \\ z = 8t + 8 \end{cases}$$

8. Sketch

$$s : \begin{cases} x = e^t + 9 \\ y = 7e^t \\ z = 8e^t + 8 \end{cases}$$

Surfaces in R^3

Identify and describe the following surfaces. You may sketch the graph but it is acceptable to just adequately describe it in words.

1. The surface $z^2 + y^2 = 9$ in R^3.
2. The surface $x^2 + x + y^2 - y + z^2 = 10$ in R^3.
3. The surface $y^2 + z^2 = x^2$ in R^3.

Vector-valued functions

1. Sketch the graph of the curve traced out by the endpoints of$\mathbf{r}(t) =< 3\sin t, 3\cos t, t >$. Make sure you indicate the orientation.

2. Sketch the graph of the curve traced out by the endpoints of $\mathbf{r}(t) =< -t, 3t\cos t, 3t\sin t >$. Make sure you indicate the orientation.

3. Sketch $\mathbf{r}(t) =< t, 3t, \tan t >$. Make sure you indicate the orientation.

4. What is the domain of $\mathbf{r}(t) =< \ln(t^2 - 1), \sqrt{-t^2 + 5}, \frac{1}{t-2} >$.

5. The vector-valued function $\mathbf{r}(t) =< t, t, \frac{t^2-1}{t-1} >$ is not defined at $t = 1$. What value of a makes the function
$$\mathbf{s}(t) = \begin{cases} < 1, 1, a > & \text{if } t = 1 \\ \mathbf{r}(t) & \text{if } t \neq 1. \end{cases}$$
continuos at $t = 1$.

6. Find t such that $\mathbf{r}(t) =< t, t, t^2 - 1 >$ and $\mathbf{r}'(t)$ are perpendicular.