

Name _____

Review

Vector Fields:

1. Plot the vector field $\mathbf{F}(x, y) = \langle x, y \rangle$.
2. Let f be a real function in three variables and $\mathbf{F} = \langle f_1(x, y, z), f_2(x, y, z), f_3(x, y, z) \rangle$ a vector field in three variables. For the following, decide whether the formula makes sense, if it does express it in term of partial derivatives, f , f_1 , f_2 and f_3 and write carefully whether is a scalar or vector quantity.
 - (a) $\nabla \cdot (\nabla f)$. (It makes sense and it is a scalar function)
 - (b) $\nabla(\nabla f)$. (It does not make sense)
 - (c) $\nabla(\nabla \mathbf{F})$. (It does not make sense)
 - (d) $\nabla \cdot \mathbf{F}$. (It makes sense and it is a scalar function)
 - (e) $\nabla \times (\nabla f)$. (It makes sense and it is a vector-valued function)
 - (f) $\nabla \times (\nabla \mathbf{F})$. (It does not make sense)
 - (g) $\nabla \cdot (\nabla \times \mathbf{F})$. (It makes sense and it is a scalar function)

Line Integrals. In the following $f(x, y, z)$ is a continuous function everywhere.

1. Let C be an oriented curve in a three dimensional space. Assume that C is described parametrically by $\langle x(t), y(t), z(t) \rangle$ for $a \leq t \leq b$, and $x(t), y(t), z(t)$ are continuous with continuous first derivatives. How do you compute the line integral of $f(x, y, z)$ with respect to the arc length ($\int_C f(x, y, z) ds$)? (i.e. State the evaluation Theorem I)
2. Let $g(x, y, z) = e^x$. Give an interpretation for $\int_C g(x, y, z) ds$, where C is the piece of the helix $\langle \cos(t), \sin(t), t \rangle$, for $0 \leq t \leq 1$.
It is the mass of the piece of helix if the density function is $g(x, y, z)$, you will solve it by computing the integral $\int_0^1 e^{\cos(x)} \sqrt{\sin(x)^2 + \cos(x)^2 + 1} dt$.
3. Is $\int_C f(x, y, z) ds = \int_{-C} f(x, y, z) ds$?
Yes.
4. If $f(x, y, z) = 1$, give the interpretation of $\int_C f(x, y, z) ds$.
It is the arc length of C .
5. Let C be an oriented curve in a three dimensional space. Assume that C is described parametrically by $\langle x(t), y(t), z(t) \rangle$ for $a \leq t \leq b$, and $x(t), y(t), z(t)$ are continuous with continuous first derivatives. How do you compute the line integral of $f(x, y, z)$ with respect to x ($\int_C f(x, y, z) dx$)? (i.e. state the evaluation Theorem II) What about $\int_C f(x, y, z) dy$ and $\int_C f(x, y, z) dz$?
6. Is $\int_C f(x, y, z) dx = \int_{-C} f(x, y, z) dx$?
No.

7. What is it $\int_C \mathbf{F} \cdot d\mathbf{r}$ in terms of line integrals if $\mathbf{F} = \langle f_1(x, y, z), f_2(x, y, z), f_3(x, y, z) \rangle$ is a vector field?

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C f_1 dx + f_2 dy + f_3 dz.$$

8. Give an interpretation of $\int_C \mathbf{F} \cdot d\mathbf{r}$ if $\mathbf{F} = m\mathbf{g}$, where m is a mass and \mathbf{g} is the gravity.
It is the work done by the gravity.

Conservative fields, for two dimensional vector fields: In the following let $\mathbf{F}(x, y) = \langle M(x, y), N(x, y) \rangle$ be a vector field with continuous partial derivatives over R^2 .

Mark as FALSE or TRUE. If the statement is false, write the correct version.

1. If \mathbf{F} is a conservative field then $\int_C M(x, y) ds = 0$ for all closed path C . TRUE.
2. If \mathbf{F} is a conservative field then $\int_C \mathbf{F} d\mathbf{r}$ does not depend on the path C but just on the initial and final point. TRUE.
3. If $M_x(x, y) = N_y(x, y)$ then \mathbf{F} is conservative. FALSE, it is $M_y(x, y) = N_x(x, y)$.
4. If \mathbf{F} is conservative then $M_y(x, y) = N_x(x, y)$. TRUE.
5. If \mathbf{F} is conservative then there exists a function in two variables such that $\nabla f(x, y) = \mathbf{F}$. TRUE.

Surface Integrals.

1. State the Evaluation Theorem for surface integrals with all the assumptions.
2. Find the surface area of the portion of the cone $z = \sqrt{x^2 + y^2}$ under the plane $z = 4$.
Compute $\iint_R 1 \sqrt{\left(\frac{2x}{2\sqrt{x^2+y^2}}\right)^2 + \left(\frac{2y}{2\sqrt{x^2+y^2}}\right)^2 + 1} dR$, where R is the circle of radius 4 and center the origin in the xy -plane.
3. Compute $\iint_S z^2 dS$, where S is the portion of the cone $z = \sqrt{x^2 + y^2}$ above the rectangle $0 \leq x \leq 2, -1 \leq y \leq 2$.
Compute $\iint_R (\sqrt{x^2 + y^2})^2 \sqrt{\left(\frac{2x}{2\sqrt{x^2+y^2}}\right)^2 + \left(\frac{2y}{2\sqrt{x^2+y^2}}\right)^2 + 1} dR$, where R is the rectangle.

Green's Theorem.

1. State Green's Theorem, with all the assumptions.
2. What is the orientation of the curve in Green's Theorem.
3. Let $\mathbf{F}(x, y) = \langle \frac{x}{y}, y \rangle$. Can you apply Green's theorem to compute $\int_C \mathbf{F} d\mathbf{r}$, where C is the circle with center the origin and radius 3? Explain your answer.
No, since the vector field is not continuous on the region.
4. Let $\mathbf{F} = \langle y + e^{\sqrt{x}}, 2x + \cos(y^2) \rangle$. Compute $\int_C \mathbf{F} \cdot \mathbf{r}$, where C is the triangle with vertices $(-1, 0), (1, 0), (0, 2)$.
Compute $\int_0^2 \int_{2x+2}^{-2x+2} (1-2) dy dx$

The Divergence Theorem

1. State The Divergence Theorem with all the assumptions.
2. Compute the flux of the vector field $\mathbf{F} = \langle x^3, y^3, z^3 \rangle$ through the sphere of radius 3.
Compute

Stokes' Theorem.

1. State Stokes' Theorem with all the assumptions. How it is different from Green's Theorem?
Green's Theorem works for 2-dimensional vector fields.
2. Use Stoke's Theorem to compute $\int_C \mathbf{F} \cdot \mathbf{r}$ where $\mathbf{F}(x, y, z) = \langle e^x, e^{-x}, e^z \rangle$ and C is the boundary of the part of the plane $2x + 2y + 2z = 2$ that lies in the first octant.
Compute $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS$, where S is the plane that lies in the first octant ($\nabla \times \mathbf{F} = \langle 0, 0, -e^{-x} \rangle$ and $\mathbf{n} = \frac{1}{\sqrt{12}} \langle 2, 2, 2 \rangle$). This makes the integral to solve to be $\int_0^1 \int_0^{1-x} -2xe^{-x} dS = \iint_R -\frac{2}{\sqrt{12}} xe^{-x} \sqrt{1+1+1} dydx$.
3. Use Stoke's Theorem to compute $\int_C \mathbf{F} \cdot \mathbf{r}$ where $\mathbf{F}(x, y, z) = \langle x + y^2, y + z^2, z + x^2 \rangle$ where C is the triangle with vertices $(1, 0, 0), (0, 1, 0), (0, 0, 1)$.
Same of above but now ($\nabla \times \mathbf{F} = \langle -2z, 2x, 0 \rangle$ the plane is the one identified by the three points, $x + y + z = 1$, hey it it is the same plane.