Mathematics 208
Fall 2006 Name ____________________

Quiz 1: Solutions

1. Give a vector $\mathbf{v}$ of length 6 that has opposite direction of $\mathbf{v} =< 1, 2, 1 >$.
$||\mathbf{v}|| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$. $\mathbf{w} = -6/\sqrt{6} < 1, 2, 1 >= -\sqrt{6} < 1, 2, 1 >$.

2. By using vectors, decide whether the three points $P(2, 9, 1), Q(0, 0, 1)$ and $R(8, 1, 5)$ are colinear (i.e. they belong to the same line).
Consider the vectors $\overline{QP} =< 2, 9, 0 >$ and $\overline{QR} =< 8, 1, 4 >$. For the three points to be colinear we need the two vectors to be parallel. For it there must be a scalar c such that $c\overline{QR} = \overline{QP}$. In particular we need to find a c such that $c8 = 2, c1 = 9, c4 = 0$. Such c doesn't exists and therefore the three points are not in the same line.

3. Mark as true of false the following statements and if they are false find a countexample.

   (a) If $\mathbf{v} =< x, y, z >$ is a vector in the 3-dimensional space then $\mathbf{v} \cdot \mathbf{v}$ is equal to the magnitude of $\mathbf{v}$.
   FALSE: $\mathbf{v} \cdot \mathbf{v} = x^2 + y^2 + z^2 = ||\mathbf{v}||^2$.

   (b) Let $\mathbf{v} =< 0, 1, 1 >$ and $\mathbf{w} =< x, y, z >$ be vectors in the 3-dimensional space. If $\mathbf{w} \cdot \mathbf{v} = 0$ then $\mathbf{w} = \mathbf{0}$ (the zero vector).
   FALSE: Let $\mathbf{v} =< 1, 0, 0 >$, then $\mathbf{w} \cdot \mathbf{v} = 0$ but $\mathbf{v} \neq \mathbf{0}$.

   (c) Let $\mathbf{w}$, $\mathbf{v}$ and $\mathbf{z}$ be three vectors in the 3-dimensional space. If $\mathbf{w} \cdot \mathbf{v} = \mathbf{w} \cdot \mathbf{z}$ then $\mathbf{v} = \mathbf{z}$.
   FALSE: The same example of above works with $\mathbf{z}$ being the zero vector.