

**Basic rules:** This is your first take home quiz. You are allowed to use your book but you cannot talk to anybody else beside me. The quiz is due Friday after Thanksgiving break. Good luck!

1. (10 points) Mark as true or false the following statements:
  - (a) To compute the mass of a rope of density  $\rho(x, y, z)$  we can use the integral  $\int_C \rho(x, y, z) ds$ , where  $C$  is the line describing the curve in the space.
  - (b) The flow lines of a vector field at any point are tangent to the vector field.
  - (c) Green's Theorem helps in computing the mass as in point (a).
  - (d) Green's Theorem can be used to compute the work of the vector field  $\mathbf{F}(x, y) = \langle x/y, \frac{y}{x+y} \rangle$  on an object which moves along the circle of radius one and center the origin.
  - (e) Green's Theorem can be used only if the vector field is conservative.
2. (10 points) Briefly explain the difference between  $\int_C f(x, y, z) ds$  and  $\int_C \mathbf{F} d\mathbf{r}$ , where  $f$  is a three dimensional real function,  $\mathbf{F}$  is a vector field and  $C$  is a curve in the space.

Explain what does it means for a vector field to have  $\text{div}\mathbf{F}(P) = 0$  and  $\text{curl}\mathbf{F}(P) = 0$  at a point  $P$ .

3. (10 points) Write the surface of the top half sphere as a surface integral.

Write the volume of the top half sphere as a

- (a) a double integral;
- (b) a triple integral.

4. (10 points) Consider the vector field given by  $\mathbf{F}(x, y) = \langle x^2 + 1, 2 \rangle$ .

(a) Sketch the vector field.

(b) Is this vector field conservative?

(c) Find the flow lines for the given vector field.

5. Compute the work of a force  $\mathbf{F}(x, y) = 2x, 2y >$  done along the portion of parabola  $y = x^2$  from  $(0, 0)$  to  $(1, 1)$ .
6. (10 points) Use Green's Theorem to evaluate the integral  $\int_C \mathbf{F} d\mathbf{r}$ , where  $\mathbf{F}$  is a vector field given by  $\langle y + e^{\sqrt{x}}, 2x + \cos(y^2) \rangle$ , and  $C$  is the boundary of the region enclosed by the parabolas  $y = x^2$  and  $y = x$  and  $C$  is oriented negatively.
7. (10 points) Consider the vector field given by  $\mathbf{F}(x, y) = \langle x^2 + 1, y^3 - 3y + 2 \rangle$ .
- (a) This vector field is conservative. Compute the function its potential function  $f$ .
- (b) Assume that  $C$  is a piecewise-smooth curve, with initial point  $(0, 0)$  and final point  $(3, 1)$ . Using the potential function of  $\mathbf{F}$  compute  $\int_C \mathbf{F} d\mathbf{r}$ , where  $C$  is any curve from  $(0, 0)$  to  $(3, 1)$ .

8. (10 points) By using the  $\text{curl}\mathbf{F}$  decide if the vector field  $\langle x^2 + 1, y, 1 - 3z \rangle$  is conservative and/or incompressible.

9. (10 points) As an application of Green's Theorem, use a line integral to compute the area enclosed by the parabola  $y = x^2$  and the line  $y = 4$ .

10. (10 points) Use spherical coordinates to compute  $\int \int \int_Q \sqrt{x^2 + y^2} e^z dV$ , where  $Q$  is the region inside  $x^2 + y^2 = 1$  and between  $z = (\sqrt{x^2 + y^2})^3$  and  $z = 0$ .