THE JOY OF NUMBERS

1. Congruence: a first look

We began class with the following definition:

(1.1) **Definition.** Let a, b, and m be integers, we say that a is congruent to b modulo (mod) m if the difference $a-b$ is divisible by m. We write $a \equiv b \mod m$.

Sara, Kelsey and Emily decided that it was better to write down the definition all in mathematical terms:
a is congruent to b if and only if there exists an integer k such that $km = a - b$ We run some experiments.

a	b such that $b \equiv a \mod 4$	a	b such that $b \equiv a \mod 5$
0	-4, 4, 8, 12, 16, 20 ...	0	-5, 5, 10, 15, 20, 25 ...
1	-3, 5, 9, 13, 17, 21, ...	1	-4, 6, 11, 16, 21, 26, ...
2	-2, 6, 10, 14, 18, 22, ...	2	-3, 7, 12, 17, 22, 27, ...
3	-1, 7, 11, 15, 19, 23, ...	3	-2, 8, 13, 18, 23, 28, ...
4	0, 8, 12, 16, 20, 24, ...	4	-1, 9, 14,19, 24, 29, ...
5	1, 9, 13, 17, 21, 25,...	5	0, 5, 10, 15, 20, 25,...
6	2, 10, 14, 18, 22, 26,...	6	1 ,6, 11, 16, 21, 26,...
7	3, 11, 15, 19, 23, 27,...	7	2, 7, 12,17, 22, 27,...
8	4, 12, 16, 20, 24, 28,...	8	3, 8, 13, 18, 23, 28,...
9	5, 13, 17, 21, 25, 29, ...	9	4, 9, 14, 19, 24, 29, ...
10	6, 14, 18, 22, 26, 30, ...	10	5, 10, 15, 20, 25, 30, ...

We then asked ourself if there is some pattern, and Eric came up with the following:

(1.2) **Conjecture.** If x is an element of row-(y), then row-(x)=row-(y).

Since the term "row" did not look very much as a math term, we notice that what it is in row-(y) is all the integers that are congruent to y modulo 5. We decide to denote that as a set $S_y = \{a \in \mathbb{Z} \mid a \equiv x \mod 5\}$. With this notation, we came up with a lot of conjectures

(1.3) **Conjecture.** (1) If x is an element of S_y, then $S_x = S_y$.
(2) There are at most m different sets S_x.
(3) (Kevin) The smallest positive integer in S_x is the remainder of x divided by m.

(1.4) **Exercise.** For next time, prove the above conjectures. Also remember that we need to prove the proposition that finishes the "Fundamental Theorem of Arithmetic".

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