

INTEGRAL CLOSURE OF RINGS AND IDEALS

MWF 1:30-2:30 Avery 111.

Instructor Information:

Name: Janet Striuli
Office: 338 Avery Hall
Phone: 472-7253
email: jstriuli2@math.unl.edu
Instructor Web Page: <http://www.math.unl.edu/~jstriuli2>
Office Hours: MWF 11:30-12:30 or drop in anytime.

Text:

Irena Swanson and Craig Huneke,
Integral Closure of Ideals, Rings and Modules.
Cambridge Univeristy Press.

Introduction: The integral closure of an ideal is very classical notion in algebraic geometry and commutative algebra. It appears in many problems: from the growth of powers of an ideal to the growth of Betti's number.

We will start the study of integral closure from the definition and we will aim to Rees's characterization of analytically unramified rings and the Briançon-Skoda Theorem.

I will try to make the class as self-contained as possible. People should be familiar with the content of *Introduction to Commutative Algebra*, by M.F. Atiyah and I.G. Macdonald. On the other hand we will spend the first two weeks of classes reviewing the content of chapter 5 of such book: integral closure of rings.

The goal of the class is to learn some classical tools in commutative algebra and to get an exposure to some problems that are still open (and of course to have a lot of fun).

Homework and Grading: There will be two set of exercises and both of them need to be approved to pass the class.

Contents: The following is a tentative list of things we will study:

1. Definition of integral closure of an ideal and basic properties.
2. Integral closure of rings and basic properties
3. Lying over, Incomparability, Going up and the Theorem on dimension.
4. Integral closure of rings and grading.
5. The Rees algebra, the graded algebra, the fiber cone and other basic properties of the integral closure of an ideal.
6. The analytic spread of an ideal.
7. Some application of the integral closure of a ring to homological algebra.

8. Integral closure of monomial ideals and power of ideals.
9. Analytically unramified rings.
10. Reductions.
11. Briancon-Skoda Theorem.
12. Valuations (if time permitted)

Of course we will adjust the contents according to the need of the class.