(1) June '04 #6 Let $f : (0,1) \to \mathbb{R}$ be a function.

Use the definition only to show that $\lim_{x \to a} \frac{x^2}{1-x} = \frac{a^2}{1-a}$ for every $a \in (0,1)$.

(2) June '02 #1

Consider the series $\sum_{k=1}^{\infty} \frac{1}{1+z^k}$, where $z \in \mathbb{C}$. Prove that the series diverges for all $|z| \leq 1$; and the series converges absolutely for all $|z| > 1$.

(3) Jan '03 #1 Suppose that $f : [0,1] \to \mathbb{R}$ is continuous, differentiable on $(0,1)$, $f(0) = f(1) = 0$ and there is $x \in (0,1)$ with $f(x) = 1$. Prove that there is some $c \in (0,1)$ with $|f'(c)| > 2$.

(4) Jan '06 #2

Assume that $f$ is differentiable at $a$. Evaluate

$$\lim_{x \to a} \frac{a^n f(x) - x^n f(a)}{x - a} \qquad (n \in \mathbb{N})$$

(5) June '05 #3

(a) Let $(a_n)_{n=1}^{\infty}$ be a sequence of real numbers such that $\sum_{n=1}^{\infty} a_n$ converges conditionally. Let $p_1, p_2, \ldots$ denote the positive terms of $\sum_{n=1}^{\infty}$ in the same order in which they occur; similarly let $q_1, q_2, \ldots$ denote the negative terms of $\sum_{n=1}^{\infty} a_n$ in the same order in which they occur. Use the definition of convergent series to show that both $\sum_{n=1}^{\infty} p_n$ and $\sum_{n=1}^{\infty} q_n$ diverge.

(b) If the series $\sum_{n=0}^{\infty} a_n$ converges conditionally, show that the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n x^n$ is 1.