

Math 826 Optional Review Questions

1. Suppose f_n and f are functions from A to B and $f_n \rightarrow f$ uniformly on A . Suppose further that $g : B \rightarrow C$ is uniformly continuous. Prove that $g \circ f_n \rightarrow g \circ f$ uniformly on A . (Pay attention to why uniform continuity is needed!)
2. Let $f_n(x) = 1/x^n$. Prove that $f_n(x) \rightarrow 0$ uniformly on $[a, \infty)$ for each $a > 1$, but not on $(1, \infty)$.
3. Does $f_n(x) = x(1-x)^n$ converge uniformly on $[0, 1]$?
4. Suppose $f_n, g_n : X \rightarrow \mathbb{R}$ converge uniformly to f and g respectively. Does $f_n(x)g_n(x)$ converge uniformly?