Math 826 Optional Review Questions

1. Suppose $f_n$ and $f$ are functions from $A$ to $B$ and $f_n \to f$ uniformly on $A$. Suppose further that $g : B \to C$ is uniformly continuous. Prove that $g \circ f_n \to g \circ f$ uniformly on $A$. (Pay attention to why uniform continuity is needed!)

2. Let $f_n(x) = 1/x^n$. Prove that $f_n(x) \to 0$ uniformly on $[a, \infty)$ for each $a > 1$, but not on $(1, \infty)$.

3. Does $f_n(x) = x(1 - x)^n$ converge uniformly on $[0, 1]$?

4. Suppose $f_n, g_n : X \to \mathbb{R}$ converge uniformly to $f$ and $g$ respectively. Does $f_n(x)g_n(x)$ converge uniformly?