

Math 826 Homework 5
Assigned: 3/25/2009, Due: 4/6/2009

1. Let $K := \{1/n : n \in \mathbb{N}\} \cup \{0\}$. Prove that K is compact.

2. Let K be a compact set. Prove that there are $a, b \in K$ such that

$$d(a, b) = \sup\{d(x, y) : x, y \in K\}$$

3. Let $S := \{(0, y) : -1 \leq y \leq 1\} \cup \{(x, \sin(1/x)) : x > 0\}$. Prove that S is connected but not path connected.

4. Let C_1, C_2 be connected, and $C_1 \cap C_2 \neq \emptyset$. Prove that $C_1 \cup C_2$ is connected.