Math 826 Homework 2

Assigned: 1/21/2009, Due: 1/28/2009

1. Suppose that f is Riemann integrable on $[a, b]$. Prove that f^2 is also Riemann integrable on $[a, b]$. Hence, by considering $f + g$ and using the first part, show that if f and g are Riemann integrable on $[a, b]$, then so is fg.

2. Suppose that $0 < a_k < 1$, that a_k is strictly decreasing, and that $a_k \longrightarrow 0$. Suppose f is bounded on $[0, 1]$ and is continuous on each open interval (a_{k+1}, a_k). Prove that f is Riemann integrable on $[0, 1]$.

3. Let f be the function defined on $\mathbb{R}$ by

$$f(x) := \begin{cases} \frac{1}{n} & \text{if } x = \frac{m}{n} \text{ where } m \in \mathbb{Z}, n \in \mathbb{N} \text{ and } gcd(m, n) = 1 \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$$

   Prove that f is Riemann integrable on $[0, 1]$.

4. Suppose that f is differentiable on (a, b) and that its derivative is bounded. Prove that f is uniformly continuous.