Math 825 Homework 6

Assigned: 12/01/2008, Due: 12/10/2008

1. Suppose that $f : D \subseteq \mathbb{R}^M \longrightarrow \mathbb{R}^N$, that x_0 is a limit point of D, and that $\lim_{x \longrightarrow x_0} f(x)$ exists. Prove that there exists a $\delta > 0$ and $M \in \mathbb{R}$ such that $\|f(x)\| < M$ for all $0 < \|x - x_0\| < \delta$.

2. In class, we proved Proposition 5.3 using Proposition 5.1. Now, prove 5.3 (ii) directly, using only the $\epsilon - \delta$ definition of limit: Let $f, g : D \subseteq \mathbb{R}^M \longrightarrow \mathbb{R}$ and let x_0 be a limit point of D. If $\lim_{x \longrightarrow x_0} f(x) = F$ and $\lim_{x \longrightarrow x_0} g(x) = G$, prove that $\lim_{x \longrightarrow x_0} f(x)g(x) = FG$

3. Prove Proposition 5.4: Suppose $f : D \subseteq \mathbb{R}^M \longrightarrow \mathbb{R}^N$ and x_0 is a limit point of D. Then $f(x) \to L$ as $x \to x_0$ iff $\pi_i(f(x)) \to \pi_i(L)$ as $x \to x_0$ for each $i = 1, 2, \ldots, N$.

4. Suppose $f : D \subseteq \mathbb{R} \longrightarrow \mathbb{R}^N$ and that x_0 is both a left limit point of D and a right limit point of D. Prove that $\lim_{x \longrightarrow x_0} f(x) = F$ iff $\lim_{x \longrightarrow x_0+} f(x) = F$ and $\lim_{x \longrightarrow x_0-} f(x) = F$

5. (a) Show that $m(x, y) = \max\{x, y\}$ is continuous on $\mathbb{R}^2$. (b) Let f and g be continuous from a set $D \subseteq \mathbb{R}^N$ into $\mathbb{R}$. Prove that $\max\{f(x), g(x)\}$ is continuous on D.