

Math 825 Homework 5  
Assigned: 10/27/2008, Due: 11/5/2008

1. Determine the convergence/divergence of the following series:

$$\sum_{n=1}^{\infty} \frac{5n}{n^2 + 1}, \quad \sum_{n=1}^{\infty} \frac{1}{1 + n^2}, \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/n}}, \quad \sum_{n=1}^{\infty} \frac{n(n-1)}{10^n}$$

2. If  $\sum_{k=1}^{\infty} a_k^2$  and  $\sum_{k=1}^{\infty} b_k^2$  converge, prove that  $\sum_{k=1}^{\infty} a_k b_k$  converges.
3. Suppose  $a_n \rightarrow 0$ . Show that there exists a subsequence  $a_{n_k}$  such that  $\sum_{k=1}^{\infty} a_{n_k}$  converges.
4. Given a series  $\sum_{n=1}^{\infty} a_n$  and a strictly increasing sequence of natural numbers  $n_k$  starting with  $n_1 = 1$ , let

$$c_k := \sum_{n_k \leq i < n_{k+1}} a_i$$

Show that, if  $\sum_{n=1}^{\infty} a_n$  converges, then  $\sum_{k=1}^{\infty} c_k$  converges to the same sum. Is the converse true?