

Math 825 Homework 4

Assigned: 10/8/2008, Due: 10/20/2008-ish

1. Let Ω be the set of all infinite sequences of 0's and 1's. More precisely, $\omega \in \Omega$ if ω is a sequence $\omega = (\omega_k)_{k=1}^{\infty}$ where $\omega_k \in \{0, 1\}$ for all k . Prove that Ω is uncountable.
2. Let Ω_p be the set of all *periodic* sequences in Ω . That is, a sequence $\omega \in \Omega$ belongs to Ω_p if and only if there is an k_0 such that $\omega_k = \omega_{k_0+k}$ for all k . Prove that Ω_p is countable.
3. Prove that $|\mathbb{R} \times \mathbb{R}| = |\mathbb{R}|$.
4. It follows immediately from Proposition 2.3 that the set of *all* subsets of \mathbb{N} is uncountable. Prove that the set of all *finite* subsets of \mathbb{N} is countable.