Math 825 Homework 3
Assigned: 9/24/2008, Due: 10/6/2008-ish

1. (*June 2001 Qualifier*) Let (x_n) be a bounded sequence in $\mathbb{R}$. Prove that if every convergent subsequence of (x_n) converges to the same limit, them $\lim x_n$ exists.

2. Suppose that every subsequence of (x_n) has a subsequence that converges to x. Prove that (x_n) converges to x.

3. Let (x_n) and (y_n) be bounded real sequences. Prove that

$$\limsup x_n + y_n \leq \limsup x_n + \limsup y_n$$

   and give an example to show that strict inequality is possible.

4. Let (x_n) be a sequence of real numbers. Say that $x \in \overline{\mathbb{R}}$ is a *subsequential limit point* of (x_n) is x_n has a subsequence that converges to x. Let S be the set of all subsequential limit points of $x(n)$. Prove that

$$\sup S = \limsup x_n \text{ and } \inf S = \liminf x_n$$