

Math 825 Homework 2 – Assigned: 9/9/2008, Due: 9/17/2008-ish

1. Let $A, B \subseteq \mathbb{R}$ be non empty and bounded above. Prove that

$$\sup(A + B) = \sup(A) + \sup(B)$$

where we define

$$A + B := \{a + b : a \in A, b \in B\}$$

2. Let $\emptyset \neq A \subseteq \mathbb{R}$ be bounded below. Prove that

$$\inf(A) = -\sup(-A)$$

where we define

$$-A := \{-a : a \in A\}$$

3. Let $S = \{q \in \mathbb{Q} : q^2 < 2\}$ and let

$$B := \{a \in \mathbb{Q} : a \text{ is an upper bound for } S\}$$

Prove that B is non-empty but that it has no smallest element. (This question shows that, although \mathbb{Q} has the same rules of arithmetic and inequalities as \mathbb{R} , it does not have the Least Upper Bound Property. There are non-empty subsets of \mathbb{Q} which have upper bounds but such that the set of upper bounds in \mathbb{Q} has no smallest element.)