

Math 825 Homework 1 – Assigned: 8/27/2008, Due: 9/3/2008

1. Let  $(x_n)_{n=1}^{\infty}$  be a sequence and let  $x \in \mathbb{R}$ .

(a) Suppose you know that:

$$\forall \epsilon > 0 \exists N \in \mathbb{N} \text{ such that } |x_N - x| < \epsilon$$

Does this imply that  $x_n \rightarrow x$ ? Prove this, or else provide a counter-example.

(b) Suppose you know that:

$$\forall \epsilon > 0 \exists N \in \mathbb{N} \text{ such that } \forall n \geq N |x_n - x| < 2\epsilon$$

Does this imply that  $x_n \rightarrow x$ ? Prove this, or else provide a counter-example.

(c) Finally, suppose you know that:

$$\forall \epsilon > 0 \exists N \in \mathbb{N} \text{ such that } \forall n \geq N |x_n - x| \leq \epsilon$$

Does this imply that  $x_n \rightarrow x$ ? Prove this, or else provide a counter-example.

2. Prove the so-called “Reverse Triangle Inequality,” that for any  $a, b \in \mathbb{R}$ ,

$$\left| |a| - |b| \right| \leq |a - b|$$

3. Suppose that  $x_n \rightarrow x$ . Prove that  $|x_n| \rightarrow |x|$ . Is the converse true?

4. Suppose  $x_n \rightarrow x$  and  $y_n \rightarrow y$  and that  $x_n \leq y_n$  for all  $n$ . Prove that  $x \leq y$ .