MATH 107-253 Recitation 4-5 Solutions

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p. 130 #51: Find the indefinite integral of $\int \left(t\sqrt{t} + \frac{1}{t\sqrt{t}} \right) \, dt$**.**

First, because the integral is the sum of two other integrals, let's split it up:

$$\int \left(t\sqrt{t} + \frac{1}{t\sqrt{t}} \right) \, dt = \int t\sqrt{t} \, dt + \int \frac{1}{t\sqrt{t}} \, dx$$

Now let's get the integral into a form we're more comfortable with. Remember that $\sqrt{x} = x^{\frac{1}{2}}$ and $x^a \times x^b = x^{a+b}$:

$$\int t\sqrt{t} \, dt + \int \frac{1}{t\sqrt{t}} \, dx = \int t^{\frac{3}{2}} \, dt + \int \frac{1}{t^{\frac{3}{2}}} \, dx$$

Also remember that $\frac{1}{x} = x^{-1}$ so

$$\int t^{\frac{3}{2}} \, dt + \int \frac{1}{t^{\frac{3}{2}}} \, dx = \int t^{\frac{3}{2}} \, dt + \int t^{-\frac{3}{2}} \, dx$$

Now both of these are in a form we recognize!

$$\int t^{\frac{3}{2}} \, dt + \int t^{-\frac{3}{2}} \, dx = \frac{1}{\frac{5}{2}} t^{\frac{5}{2}} + C_1 + \frac{1}{-\frac{1}{2}} t^{-\frac{1}{2}} + C_2$$

Each integral has a separate constant, but since we are adding them to solve our original integral we can combine them into one constant. We can also simplfy our fractions and get

$$\frac{1}{\frac{5}{2}} t^{\frac{5}{2}} + C_1 + \frac{1}{-\frac{1}{2}} t^{-\frac{1}{2}} + C_2 = \frac{2}{5} t^{\frac{5}{2}} - 2t^{-\frac{1}{2}} + C$$

or if you prefer

$$\frac{2}{5} t^{\frac{5}{2}} - 2t^{-\frac{1}{2}} + C = \frac{2}{5} t^{\frac{5}{2}} - \frac{2}{\sqrt{t}} + C.$$

p. 338 #30: Find $\displaystyle\int \frac{e^t+1}{e^t+t}\,dt$. Check your answer by differentiating.

Note that the derivative of the denominator is the numerator! Looks like a good candidate for u-substitution.

Let $u = e^t + t$. Then $\frac{du}{dt} = e^t + 1$ and $dt = \frac{du}{e^t+1}$. So

$$\begin{aligned}
\int \frac{e^t+1}{e^t+1}\,dt &= \in \frac{\cancel{e^t+1}}{u}\frac{du}{\cancel{e^t+1}} \\
&= \int \frac{1}{u}\,du \\
&= \ln|u| + C \\
&= \ln|e^t+t| + C
\end{aligned}$$

Finally,

$$\frac{d}{dx}[\ln|e^t+t|] = \frac{1}{e^t+t}\cdot\frac{d}{dx}[e^t+t] = \frac{1}{e^t+t}\cdot e^t + 1 = \frac{e^t+1}{e^t+t}$$

p. 338 #40: Find $\displaystyle\int \frac{x\cos(x^2)}{\sqrt{\sin(x^2)}}\,dt$. Check your answer by differentiating.

The denominator is certainly the complicated part of the expression, but if we're going to use u-substitution should we set $u = x^2, u = \sin(x^2)$ or $u = \sqrt{\sin(x^2)}$? If we use $u = x^2$ we'll have a x stuck in the numerator so let's try $u = \sin(x^2)$. Then $\frac{du}{dx} = \cos(x^2)\cdot 2x$ so $dx = \frac{du}{2x\cos(x^2)}$. Then

$$\begin{aligned}
\int \frac{x\cos(x^2)}{\sqrt{\sin(x^2)}}\,dx &= \frac{\cancel{x\cos(x^2)}}{\sqrt{u}}\,\frac{du}{2\cdot\cancel{x\cos(x^2)}} \\
&= \int \frac{1}{2\sqrt{u}}\,du \\
&= \frac{1}{2}\int u^{-1/2}\,du \\
&= \frac{1}{2}\frac{u^{1/2}}{\frac{1}{2}} + C \\
&= \sqrt{u} + C \\
&= \sqrt{\sin(x^2)} + C
\end{aligned}$$

Finally,

$$\frac{d}{dx}[\sqrt{\sin(x^2)}] = \frac{1}{2}\cdot\frac{1}{\sqrt{\sin(x^2)}}\cdot\frac{d}{dx}[\sin(x^2)] = \frac{1}{2}\cdot\frac{1}{\sqrt{\sin(x^2)}}\cdot\cos(x^2)\cdot\frac{d}{dx}[x^2] = \frac{1}{2}\cdot\frac{1}{\sqrt{\sin(x^2)}}\cdot\cos(x^2)\cdot 2x = \frac{x\cos(x^2)}{\sqrt{\sin(x^2)}}$$

Note that because of a typo in the problem, we used dx instead of dt. However, if the problem really asked for the integral with respect to dt, then the answer would just be

$$\frac{x\cos(x^2)}{\sqrt{\sin(x^2)}}\cdot t + C$$

because we could consider anything with x to be a constant!

p. 338 #74: Find $\int (z+2)\sqrt{1-z}\ dz$.

Let's try $u = 1 - z$ to simplify the square root a little bit. Then $\frac{du}{dz} = -1$ so $dz = -du$. Plugging back in, we get

$$\int (z+2)\sqrt{u}(-du).$$

Notice that the z didn't cancel out. This might mean we need a different choice for u, but nothing stands out. Instead, we can solve for z in terms of u. Since $u = 1 - z$, we know $u - 1 = -z$ or $z = 1 - u$. Then we can instead solve

$$-\int ((1-u)+2)\sqrt{u}\ du.$$

Now this is in a form we can solve:

$$\begin{aligned}
-\int ((1-u)+2)\sqrt{u}\ du &= -\int (3-u)u^{1/2}\ du \\
&= -\left(\int 3u^{1/2}\ du + \int -u^{3/2}\ du\right) \\
&= -\left(3\frac{u^{3/2}}{\frac{3}{2}} - \frac{u^{5/2}}{\frac{5}{2}}\right) + C \\
&= -2u^{3/2} + \frac{2}{5}u^{5/2} + C \\
&= \frac{2}{5}(1-z)^{5/2} - 2(1-z)^{3/2} + C
\end{aligned}$$