MATH 107-153 Recitation 2Solutions

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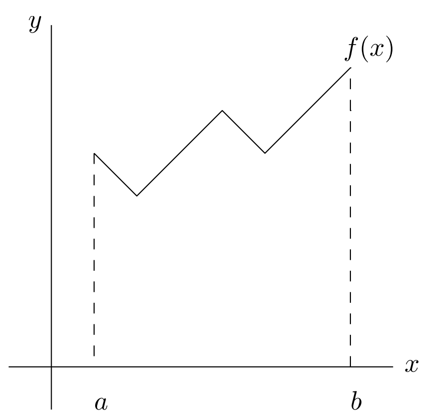
p. 269 #35: Sketch the graph of a function f (you do not need to give a formula for f) on an interval $[a, b]$ with the property that with $n = 2$ subdivisions,

$$\int_a^b f(x)\ dx < \textbf{Left-hand sum} < \textbf{Right-hand sum}$$

To answer this problem, think about how the Left- and Right-Hand Rules calculate area. For each individual rectangle, that rectangle can over- or under-estimate the area under the curve. The Left-Hand Rule overestimates when the function is decreasing (because the top of the rectangle extends over the curve) while the Right-Hand Rule overestimates when the function is increasing.

One example f is given below:

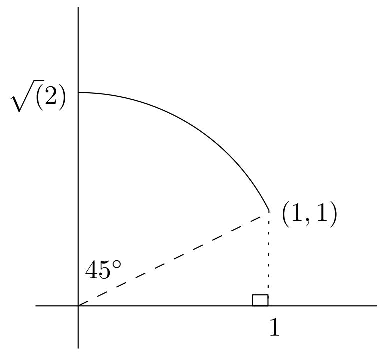


p. 365 #22:

(a) Show geometrically why $\int_0^1 \sqrt{2-x^2}\,dx = \frac{\pi}{4}+\frac{1}{2}$. [Hint: Break up the area under $y=\sqrt{2-x^2}$ from $x=0$ to $x=1$ into two pieces: a sector of a circle and a right triangle.]

(b) Approximate $\int_0^1 \sqrt{2-x^2}\,dx$ for $n=5$ using the left, right, trapezoid and midpoint rules. Compute the error in each case using the answer to part (a), and compare the errors.

(a) Sketch the graph and then draw the following line: (apologies for the not-to-scale drawing)



The triangle has base 1 and height 1 and so has area $\frac{1}{2}$. The length of the hypotenuse is $\sqrt{2}$, the same as the y-intercept and the radius of the circle. Because the angle is 45°, it has area $\frac{45}{360}\times 2\pi = \frac{\pi}{4}$. Adding these numbers together gets the correct answer.

(b)

	Estimate	Error
LEFT(5)	1.32350	-0.03810
RIGHT(5)	1.24066	0.04474
TRAP(5)	1.28209	0.00332
MID(5)	1.28705	-0.001656

3: For each cell in the chart below, put "over" if the using the approximation from that row and the type of function from that column will produce an overestimation of the integral, put "under" if it will produce an underestimation, and put "x"in the square if neither is true.

Then, for each box in which you wrote "over" or "under", justify why you put that answer. For each box you put "x", sketch the graphs of two functions such that using that approximation method produces an overestimate on one graph and an underestimate on the other.

	Increasing function	Decreasing function	Concave down	Concave up
LEFT	under	over	x	x
RIGHT	over	under	x	x
TRAP	x	x	under	over
MID	x	x	over	under

On increasing functions, LEFT estimates assume the value will stay constant at the lefthand value across δt but it increases instead. On the other hand, RIGHT estimates assume the value had been constant while in reality the function rose to that value.

The argument for decreasing functions is exactly the opposite.

Concave down functions can be increasing ($f(x) = \ln(x)$) or decreasing ($f(x) = \frac{1}{x}$ for $x < 0$). The arguments above show that LEFT and RIGHT estimations are dictated by whether a function is increasing or decreasing, so these examples show concavity does not predict error.

Concave up functions can also be increasing ($f(x) = e^x$) or decreasing ($f(x) = e^{-x}$), so the same argument holds.

For concave down functions, the trapezoid approximation takes points at the edge of the interval and assumes the function is linear between those points but the curve bends over that line, so the estimate is too small. The midpoint approximation also assumes the function acts like a line but it uses the midpoint so that line extends out over the curve creating an over approximation.

The opposite arguments hold for concave up functions.

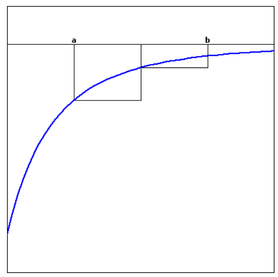
Finally, increasing functions can be concave up ($f(x) = e^x$) or concave down ($f(x) = \ln(x)$), as can decreasing functions ($f(x) = \frac{1}{x}$ for $x < 0$ and $f(x) = e^{-x}$) so slope does not affect the TRAP and MID estimations.

4: For the graph below, answer:

(a) What is n?

(b) What type of approximation is being used?

(c) Is the estimation an overestimation or an underestimation?



(a) $n = 2$

(b) Left-hand rule

(c) It is an underestimation. Even though the estimation has a greater area than the area under the curve, that area is below the x-axis so it is a larger negative number making it less than the integral. This may be counter-intuitive, but note that this function is increasing so your chart from problem three gives you the correct answer.