MATH 107-153 Recitation 1

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Agenda

- Introductions
- A note on the Philosophy of Calculus
- Policies and Expectations
- Questions and Exercises from yesterday's lecture
- Setting office hours

Introductions

1. Who are you? By what name would you like to be called?
2. What would you like to do? This might be related to your major, but I'm more interested in what you want to accomplish in life.
3. How do you feel about math and/or calculus? It's okay to be honest. I don't expect everyone to love calculus.

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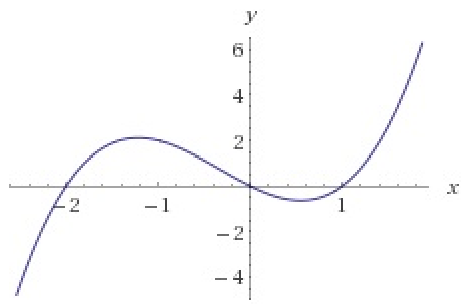
p. 269 #27:

(a) Graph $f(x) = x(x+2)(x-1)$**.**

(b) Find the total area between the graph and the x**-axis between** $x = -2$ **and** $x = 1$**.**

(c) Find $\int_{-2}^{1} f(x)\ dx$ **and interpret it in terms of areas.**

(a) First, notice we are dealing with a cubic function. Furthermore, the x^3 term is positive, so we expect the graph to tend negative for decreasing values and positive for increasing values. Because we are given the factored form, we can easily see the zeroes of the function occur at $x = 0, x = -2$ and $x = 1$. Therefore we expect the graph to look something like this:



(b) Remember that in order to calcluate the **total** area we need to take the integral of the positive parts of the graph and add the absolute value of the integral of the negative parts of the graph. Looking at what we drew above, that means we are interested in:

$$\text{Area } = \int_{-2}^{0} f(x)\ dx + \left| \int_{0}^{1} f(x)\ dx \right|$$

Let's start by finding an anti-derivative for $f(x)$. To make this easier, let's multiply out $f(x)$ to get

$$f(x) = x^3 + x^2 - 2x$$

Now it's easy to find anti-derivatives for each term and add them together:

$$F(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$$

Finally, we can use the Fundamental Theorem of Calculus and apply it to the equation:

$$\begin{aligned}
\text{Area } &= \int_{-2}^{0} f(x)\ dx + \left| \int_{0}^{1} f(x)\ dx \right| \\
&= (F(0) - F(-2)) + |(F(1) - F(0))| && \text{By the F.T.C.} \\
&= \left(0 - \left(\frac{(-2)^4}{4} + \frac{(-2)^3}{3} - (-2)^2\right)\right) + \left|\left(\frac{(1^4}{4} + \frac{1^3}{3} - 1^2\right) - 0\right| \\
&= -\left(4 - \frac{8}{3} - 4\right) + \left|\frac{1}{4} + \frac{1}{3} - 1\right| \\
&= \frac{37}{12}
\end{aligned}$$

(c) All of the hard work is done! Since we already found an anti-derivative, we can just use the Fundamental Theorem of Calculus to calculate the integral:

$$\int_{-2}^{1} f(x)\ dx = F(1) - F(-2) = -\frac{5}{12} + \frac{8}{3} = \frac{27}{12} = \frac{9}{4}$$

In terms of areas, this is the answer we get when we consider the area below the x-axis (bounded by $f(x)$) to be negative.

Suppose x measured seconds and $f(x)$ measured m/s. If we want to calculate total distance traveled, we'd want to use the answer from part (b) where all area is considered positive. If we just want to measure displacement (how far an object ended up compared to where is started) then we'd need to use the answer from part (c). Because the object's velocity went negative, it started moving back towards where it started.

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p. 269 #35: Sketch the graph of a function f (you do not need to give a formula for f) on an interval $[a, b]$ with the property that with $n = 2$ subdivisions,

$$\int_{a}^{b} f(x)\ dx < \textbf{Left-hand sum} < \textbf{Right-hand sum}$$

To answer this problem, think about how the Left- and Right-Hand Rules calculate area. For each individual rectangle, that rectangle can over- or under-estimate the area under the curve. The Left-Hand Rule overestimates when the function is decreasing (because the top of the rectangle extends over the curve) while the Right-Hand Rule overestimates when the function is increasing.

One example f is given below: