Annotated Bibliography on Polynomial Maps

Compiled by Gary Hosler Meisters

Last Update: July 10, 1997

Contents

Part I: Annotated Bibliography on Polynomial Maps

This bibliography includes the seven subjects listed below; merged into one list, alphabetical by author. Your comments and suggestions for corrections or additions are appreciated. The LATEX dvi-file is available on my World-Wide-Web Home-Page at the Web Address (URL):

http://www.math.unl.edu/~gmeister/

I. Polynomial maps of affine n-space $k^n$, for $n \geq 2$.

II. Polynomial maps of affine 2-space.

III. The Groups $GA[k^n]$ and $Aut[k[x_1, x_2, \ldots, x_n]]$.

IV. Nilpotent matrices of homogeneous polynomials.

V. Polyflows in $k^n$ and derivations on $k[x]$.

VI. Injectivity and surjectivity for maps of $k^n$.

VII. Global behavior of polynomial vector fields: Global attractors, almost periodic orbits, stability, etc.

The Markus-Yamabe Conjecture about the global asymptotic stability of a stationary point is true only on $\mathbb{R}^2$ [78, 160, 198, 274]. So now the foundation is laid and the stage is set for the more detailed investigation of this area. Indeed, it has already begun [71, 86, 87, 88, 90, 98, 129, 137, 195, 197, 273].

Part II: Contributors & Their Addresses


*Thanks to all who have helped. Please send corrections or additions to me at Dept Math & Stat, U Nebraska, Lincoln, NE 68588-0323. E-mail: gmeister@math.unl.edu  Tel: OFFICE (402) 472-7261; HOME (402) 488-1583; UNL-FAX (402) 472-8466. Copyright © 1995 & 1996 by G. H. Meisters
Part I. Annotated Bibliography on Polynomial Maps


[20] P. Ahern and F. Forstneric. One parameter automorphism groups on $\mathbb{C}^2$. Complex Variables, 27:245–268, (1995). An excellent paper about the classification of polynomial flows. Given a polynomial $g$ of $\mathbb{C}^2$ which is not conjugate to an affine aperiodic map $(x, y) \mapsto (x + 1, \beta y)$, for nonzero complex $\beta$, they find all real one parameter subgroups $\{\phi_t : t \in \mathbb{R}\}$ in the holomorphism group HolAut$\mathbb{C}^2$ whose time one map $\phi_1$ equals $g$. For affine aperiodic $g$ they find all such subgroups whose infinitesimal generator is polynomial. They also classify one parameter subgroups of the shear groups $S(2)$ and $S_1(2)$ on the plane $\mathbb{C}^2$. Relates to the earlier work of Suzuki [405] and Bass & Meisters [45], CMP 1333980 (95:13).


[22] M. A. Aizerman. Aizerman’s Conjecture: For each integer $k$, $1 \leq k \leq n$, the real nonlinear system

$$
\begin{align*}
\dot{x}_1 &= \sum_{j=1}^{n} a_{1j}x_j + f(x_k) \\
\dot{x}_i &= \sum_{j=1}^{n} a_{ij}x_j \\
&\quad (i = 2, 3, \ldots, n)
\end{align*}
$$

has the origin as a globally asymptotically stable rest point provided that $f(x)$ is continuous, $f(0) = 0$, and, for each $x \neq 0$, $\alpha < f(x)/x < \beta$ for every pair of real numbers $\alpha, \beta$ for which all the characteristic roots of the companion linear system

$$
\begin{align*}
\dot{x}_1 &= \sum_{j=1}^{n} a_{1j}x_j + ax_k \\
\dot{x}_i &= \sum_{j=1}^{n} a_{ij}x_j \\
&\quad (i = 2, 3, \ldots, n)
\end{align*}
$$

have negative real parts whenever $\alpha < a < \beta$. See Barabanov, Fitts, Funmin, Kalman, Pliss, Singh, and Yakubovich.


[27] H. Alexander. Proper holomorphic mappings in $\mathbb{C}^n$. Indiana Univ. Math. J., 26:137–146, (1977). Proves that when $n > 1$, the holomorphic automorphisms of the open unit ball $B$ of $\mathbb{C}^n$ are the only proper holomorphic maps of $B$ into itself.


[38] N. E. Barabanov. On a Problem of Kalman. Siberian Mathematical Journal, 29(3):333–341, May-June (1988). MR 89g:93077. From the Math Review [by Miklós Farkas] of Barabanov’s paper: Consider the system (1) $\dot{x} = Ax + b\varphi(\sigma)$, $\sigma = c^*x$, where $A$ is an n-by-n matrix, $b$ and $c$ are column vectors, and $\varphi$ is a scalar function. Assume that $\forall \mu \in (\alpha, \beta) \text{ the system (1) with } \varphi(\sigma) = \mu \sigma$ is asymptotically stable. By strengthening the assumptions in Aizerman’s Problem, R. E. Kalman conjectured that if $\varphi'(\sigma) \in (\alpha, \beta) \forall \sigma$, then the origin is globally asymptotically stable. Kalman’s conjecture is proved if dim $n = 3$; and a counterexample is given to prove that systems exist in dimension $n \geq 4$ which satisfy Kalman’s condition but still have a nontrivial periodic solution. End of Math Review. However, this paper contains several incorrect arguments and is notoriously unreadable. What Barabanov claims in this paper is certainly not proved there. But it did inspire the 1994 paper [50].


[54] Robert K. Brayton. What is the reference for Brayton’s paper(s) before 1983? Jürgen Moser says (letter to me dated April 13, 1983) “The Problem by Aizerman has . . . been solved, by counterexample; see V. A. Pliss and J. C. Willems; Brayton found a positive answer to a related problem”.


[61] L. A. Campbell. A Generalization of Dillen’s Corollary. Private Communication, April 15, 1993. Theorem: If det J(f) is a nonzero constant and the four polynomials that are the entries of J(f) are linearly dependent, then the two-dimensional polynomial map f is invertible.


[64] L. A. Campbell. Jacobian Pairs and Hamiltonian Flows. *J*P*X* Preprint, September 1995. Uses flows of the Hamiltonian vector field (−f_y, f_x) to obtain some new conditions for the invertibility of a polynomial map (f, g) : C^2 → C^2 when f_gy − f_y g_x = c ∈ C \{0\}.


[69] N. V. Chau. Global structure of a polynomial autonomous system on the plane. *Annales Polonici Mathematici*, to appear?, (1992). If F : R^2 → R^2 is polynomial, and ∀x det f'(x) ≠ 0, and there exists at least one vector v ≠ 0 such that 0 ∉ convex hull{F'(x)v : x ∈ R^2, |x| ≥ c > 0}, then f is injective.


[71] N. V. Chau. Global attractor of a differentiable autonomous system on the plane. *Ann. Polon. Math.*, LXII(2):143–154, (1995). Thm 1 generalizes Olech’s result [315], and Thm 2 is an interesting variation of the Markus-Yamabe Global Asymptotic Stability Jacobian Conjecture for the polynomial case, the original version of which was proved by Meisters & Olech [274]. Thm 1: If (i) f(0) = 0 and zero is a regular value of f (i.e., det f'(x) ≠ 0 at each x ∈ R^2 where f(x) = 0), (ii) ||f(x)|| > const > 0 for ||x|| > const, (iii) div f(x) ≤ 0 for ||x|| > const, and (iv) ∫_R^2 div f(x) dx < 0; then either (a) there is a trajectory with empty positive limit set which tends to a saddle point as t → −∞, or (b) x = 0 is a global attractor. Thm 2: If f is a polynomial map of R^2 into itself which satisfies (i)–(iii) of Thm 1, then either (a) or (b) of Thm 1 holds, or (c) every trajectory is either a centre, a saddle point, a closed curve, or a curve joining two saddle points. Neither of these results contains the earlier results of Olech or of Meisters & Olech because the author’s assumptions are different: Thm 1 has weaker assumptions and weaker conclusions than Olech’s 1963 result; while Thm 2 has both weaker & stronger assumptions and different conclusions than the 1988 result of Meisters & Olech. Next the author deduces two injectivity results (his Thms 3 & 4) which are now of even more interest since Pinčuk has given a counterexample to the Strong Real Jacobian Conjecture on R^2 [343]. Set I_f := {a ∈ R^2 : 0 < #f^{-1}(a) < ∞ and det f'(x) > 0 for all x ∈ f^{-1}(a))}. Thm 3: If int I_f ≠ 0 and (ii) of Thm 1 holds, then f maps f^{-1}(int I_f) one-to-one onto int I_f. Thm 4: If f is a polynomial map of R^2 into itself, det f'(x) > 0 on R^2, and (ii) of Thm 1 holds, then f is a homeomorphism of R^2. The paper ends with an example of a 1-parameter family of vector fields on R^2, namely f_µ(x_1, x_2) = (x_2 − x_1(x_1^2 + x_2^2 − µ), −x_1 − x_2(x_1^2 + x_2^2 − µ)), which exhibits four different types of behavior depending on whether µ ≤ 0, 0 < µ < √3, µ = √3, or µ > √3.


[89] B. A. Coomes. On the Torsion Part of $\mathbb{C}^{|n|}$ with respect to the Action of a Derivation. *Proc. Amer. Math. Soc.*, **123**(7):2191–2197, July (1995). If $F : \mathbb{C}^n \to \mathbb{C}^n$ is a polynomial mapping, $\det F'(x) \equiv 1$, $F(0) = 0$, and $D$ denotes the derivation associated with the vector field $V(y) := -[F'(y)]^{-1}F'(y)$; then $F$ is a polynomial automorphism iff the torsion part $T(D)$ of $\mathbb{C}^{|n|}$ is algebraically closed in $\mathbb{C}^{|n|}$. 


From Meisters’ Bibliography on Polynomial Maps


C. De Fabritiis. One-parameter groups of volume-preserving automorphisms of $\mathbb{C}^2$. (to appear).


Bo Deng. Analytic Conjugation, Global Attractor, and the Jacobian Conjecture. *T\textit{eX} preprint* (1995). Four pages, University of Nebraska-Lincoln, U.S.A. (bdeng@ode.unl.edu). Bo proved that the dilation $\lambda f$ of an analytic map $f$ of $\mathbb{C}^n$ into itself with $f(0) = 0$, $f'(0) = I$, and $|\lambda| > 1$, has an analytic conjugation to its linear part $\lambda x$ if and only if $f$ is a holomorphicmorphism (i.e., a holomorphic automorphism) of $\mathbb{C}^n$ and $x = 0$ is a global attractor for the inverse of $\lambda f$. This is a Corollary of the more general Theorem: Suppose $F : \mathbb{C}^n \to \mathbb{C}^n$ is an analytic map, $F(0) = 0$, $\det F'(0) \neq 0$, and all eigenvalues of $F'(0)$ are in the open unit disk and not resonant; then $F$ has an analytic conjugation to its linear part $F'(0)$ if $F$ is an analytic automorphism of $\mathbb{C}^n$ and the fixed point $x = 0$ is a global attractor, i.e., $\forall x \in \mathbb{C}^n, F^k(x) := F \circ F^{k-1}(x) \to 0$ as $k \to \infty$. Compare this with the theorem in the appendix of the 1988 paper [368] by Rosay & Rudin on holomorphic maps (listed in this PolyMapBib). The latter was kindly brought to our attention by Franc Forstneric, January 10, 1996. Bo Deng had not seen this earlier paper by Rosay and Rudin; and Meisters had not noticed that their theorem was relevant to what Bo Deng, Gaetano Zampieri, and I were then considering about the possible global conjugation of dilations of polynomial maps to their linear part. However, it was after seeing this paper of Bo Deng, not that of Rosay & Rudin, that Arno van den Essen and his colleagues were lead to write [78].

H. G. J. Derksen. The Kernel of a Derivation. *J. Pure Appl. Algebra*, 84:13–16, (1993). Nijmegen Report 9123 November 1991. Derksen wrote this as an undergraduate student of Arno van den Essen. Nagata and Nowicki showed in 1988 that the kernel of a derivation on $K_{\infty}[X_1, \ldots, X_n]$ is of finite type over $K_{\infty}$ if $n \leq 3$. Derksen constructs a derivation of a polynomial ring in 32 variables (using Nagata’s counterexample to Hilbert’s 14th Problem) whose kernel is not of finite type over $K_{\infty}$.


[114] L. M. Drużkowski. The Jacobian Conjecture in case of rank or corank less than three. J. Pure Appl. Algebra, 85:233–244, (1993). MR 93m:14011. Proves (I): Every complex matrix $A$ is cubic-similar to a complex matrix $D$ with the properties (1) $D = H_f(x)c$, for some vector $c$, and (2) the nilpotence index of $H_f(x) = 3[\text{diag}(Dx)]^2D$ is the same as the nilpotence index of $D$ itself. Also proves (II): If $\text{rank}(A)$ or $\text{corank}(A) := n - \text{rank}(A)$ is less than three, then $F_A := x - H_f(x)$ is tame; i.e., $F(x) - F(0)$ is a finite composition of linear automorphisms and nonlinear shears $T(x_1, \ldots, x_n) = (x_1, \ldots, x_{i-1}, x_i + f(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n), x_{i+1}, \ldots, x_n)$. In particular, every cubic-admissible matrix $A$ is cubic-similar to a nilpotent matrix $D$.


[118] L. M. Drużkowski. The Jacobian Conjecture in the Case of Non-Negative Coefficients. *To appear in Ann. Polon. Math.* (1996). Another proof of *Yu’s Theorem*: Polynomial maps are necessarily polynomials if they are of the form \( F(x) = x - H(x) \), where \( H(x) \) is cubic-homogeneous, \( \det H'(x) \equiv 1 \) (i.e., \( H'(x) \) is nilpotent \( \forall x \in \mathbb{R}^n \)), and all coefficients in \( H \) are non-negative. Moreover, under these conditions, \( \deg F^{-1} \leq (\deg F) \text{ind } F^{-1} \), where \( \text{ind } F := \max \{ \text{ind } H'(x) : x \in \mathbb{R}^n \} \).


[120] L. M. Drużkowski and K. Rusek. The Real Jacobian Conjecture for Cubic Linear Maps of Rank Two. *Universitatis Iagellonicae Acta Mathematica*, **32**:17–23, (1995). Every polynomial mapping of \( \mathbb{R}^n \) into itself of the cubic-linear form with rank of its homogeneous part \( \leq 2 \) and positive Jacobian determinant is bijective with an analytic inverse. This is interesting in light of Pińczuk’s non-injective example of a polynomial mapping of \( \mathbb{R}^n \) with positive Jacobian determinant.


[122] C. Eggermont and A. van den Essen. A Class of Triangular Derivations Having a Slice. Report **9429**, Mathematics Department, University of Nijmegen, The Netherlands, June 1994. The authors generalize to a special class of \( G_a \)-actions on \( \mathbb{C}^n \) \( (n \geq 3) \) Dennis Snow’s result that all free triangular \( G_a \)-actions on \( \mathbb{C}^3 \) are equivariantly isomorphic to \( G_a \times \mathbb{C}^2 \).


A. van den Essen. A counterexample to a conjecture of Drzu˙ zkowski and Rusek. Report 9440, Mathematics Department, University of Nijmegen, The Netherlands, October 1994. It was conjectured by Drzu˙ zkowski and Rusek that deg$F^{-1} \leq 3^n - 1$ if $F(x) = I + H(x)$ is a cubic-homogeneous polynomial automorphism of $\mathbb{C}^n$ and the Jacobian matrix $H'(x)$ has nilpotence index $p$. The authors prove that this is true if $n \leq 4$ but false if $n \geq 5$.

A. van den Essen. A counterexample to a conjecture of Meisters. Report 9441, Mathematics Department, University of Nijmegen, The Netherlands, October 1994. At the July 4th, 1994, Curacao Conference, Meisters offered $100 to the first person to give an example of a cubic-homogeneous polynomial mapping $F(x) = I + H(x)$ of $\mathbb{C}^n$ into itself with det$H'(x) = 1$ for which the complex $s$-parametered conjugation $h_s \circ F \circ h_{-s}^{-1} = s I$ fails (for $|s| \neq 1$) for polynomial automorphisms $h_s$. (See Meisters’ Curaçao paper for details and explicit examples.) Arno van den Essen shows in this paper that, while such polynomial conjugations do hold in dimensions $n \leq 3$, there are counterexamples in dimensions $n \geq 4$. On Monday, September 19, 1994, I found two e-mail messages from Arno van den Essen, dated September 16 and 19, in which he described two cubic-homogeneous counterexamples! On September 19 he FAXed to me (Meisters) a proof that $h(s, x)$ defined by the eq. $h_s \circ F \circ h_{-s}^{-1} = s I$ when $F(x) = (x_1 + p(x)x_4, x_2 - p(x)x_3, x_3 + x_4, x_4)$, where $p(x) = x_3x_1 + x_4x_2$, can not be polyno-


A. van den Essen. A Counterexample to a Conjecture of Shpilrain and Yu. Report 9639, Mathematics Dept, University of Nijmegen, Toernooiveld 6525 ED Nijmegen, The Netherlands, December (1996). A counterexample is given to a conjecture of V. Shpilrain & J.-T. Yu (in [387]): Namely, if $p \in k[X,Y]$ has a unimodular gradient (i.e., $1 \in \text{ideal in } k[X,Y]$ generated by the partial derivatives
of $p$), then van den Essen shows that $k[p]$ need not be a retract (see [91]); and then deduces, for every pair of integers $(n, m)$ with $n > m$, the falsity of the “Generalized Jacobian Conjecture” $GJC(n, m)$:

If $F : \mathbb{C}^n \to \mathbb{C}^m$ is a polynomial map such that the Jacobian matrix $F'(x)$ has maximal rank for all $x \in \mathbb{C}^n$, then $F$ has a left inverse.

[144] A. van den Essen. A Criterion to Decide if a Polynomial has a Jacobian Mate of Bounded Degree. Report 9625, Mathematics Dept, University of Nijmegen, Toernooiveld 6525 ED Nijmegen, The Netherlands, September 1996. Provides a criterion to decide whether or not, to a given polynomial $Q$ in $k[X, Y]$ of degree $n \geq 2$ and a given positive integer $s$, there corresponds a polynomial $P$ in $k[X, Y]$ of degree $\leq ns$ such that $[P, Q] := P_x Q_y - P_y Q_x = 1$. In case such a polynomial $P$ (called a Jacobian mate of $Q$) exists, it is shown how to construct it. The method uses and extends the work of Magnus [244]. This problem was treated in [72] when the mate $P$ is of degree smaller than $\text{deg}(Q)$, and in [401] when the $\text{deg}(P)$ has some fixed degree.

[145] A. van den Essen. Nilpotent Jacobian matrices with independent rows. Report 9603, University of Nijmegen, February 1996. Let $k$ be a field with characteristic zero and $n \geq 3$. Polynomial maps $H : k^n \to k^n$ are described such that $H'(x)$ is nilpotent for all $x \in k$, and $H_1, \ldots, H_n$ are linearly independent over $k$. It can happen that no iterate of $H$ is equal to zero.


[147] A. van den Essen et al. See also Adjamagbo, Derksen, and Eggermont. Nijmegen Reports.

[148] A. van den Essen and E. Hubbers. Polynomial maps with strongly nilpotent Jacobian matrix and the Jacobian Conjecture. Report 9444, Mathematics Department, University of Nijmegen, The Netherlands, 1994. To appear in Linear Algebra Appl. The conjugation equation $h_s \circ s \circ F \circ h_s^{-1} = s I$, conjectured by Deng, Meisters, and Zampieri for a one-parameter (complex $s$ off the unit circle) family of polynomial automorphisms $h_s$, is true when $F = I + H$ and $H'(x)$ is strongly nilpotent, but not in general. Now also settled (but not in this paper) is the question: Is this conjugation equation valid for polynomial $F$ in $x$ (or at least entire in $x$) automorphisms $h_s(x)$ when the mapping $x \mapsto F(x)$ is of Drzukowski’s cubic-linear type? As a reward for a counterexample, Meisters had offered $200 until February 17, 1996. In April 1996, van den Essen & Hubbers thought they had found such a counterexample, but found a mistake; nevertheless, they soon did prove the existence of such a counterexample [146] with the help of the beautiful results [192] of Gorni & Zampieri.

[149] A. van den Essen and Engelbert Hubbers. A New Class of Invertible Polynomial Maps. Report 9604, University of Nijmegen, February 1996. Presents a new large class of polynomial maps $F = X + H$ for which the Jacobian Conjecture is true. In particular $H$ does not need to be homogeneous. It is also shown that for all $H$ in this class satisfying $H(0) = 0$, the $n^{\text{th}}$ iterate $H \circ \ldots \circ H = 0$.

[150] A. van den Essen and Engelbert Hubbers. Chaotic Polynomial Automorphisms: counterexamples to several conjectures. Adv. in Appl. Math., 18(3):382–388, April 1997. Nijmegen Report 9549 (November 1995). Gives a counterexample, of the form $f(x) = x + H(x) \in \mathbb{Z}[x_1, x_2, x_3, x_4]$ where $H$ is homogeneous of degree 5, to the DMZ-conjecture (of Bo Deng, G. H. Meisters, and Gaetano Zampieri) to the effect that if $f : \mathbb{C}^n \to \mathbb{C}^n$ is a polynomial map with $f(0) = 0$ and $f'(0) = I$, then for all $\lambda > 1$, $\lambda$ large enough, there exists an analytic automorphism $h_\lambda : \mathbb{C}^n \to \mathbb{C}^n$ such that $h_\lambda \circ f \circ h_\lambda^{-1} = \lambda I$; i.e., $h_\lambda$ conjugates $f$ to its linear part. The authors show that this same example is also a counterexample to the discrete Markus-Yamabe Question of J. P. LaSalle [238], revived by Cima, Gasull, and Mañosas [80]. This paper still does not settle the cubic-linear linearization conjecture which Meisters mentioned in his 1994 Curacao paper [265]; but see the more recent papers of van den Essen [146] and Gorni & Zampieri [192] which do!

From Meisters’ Bibliography on Polynomial Maps


[162] L. Flatto. Invariants of finite reflection groups. Enseign. Math., 24:237–292, (1978). A nice account of a beautiful subject, discussed further by Walter Rudin in [372, this Bib]. A linear transformation \( L \) of \( \mathbb{C}^n \) into itself is called a reflection if it has finite period and its set of fixed points forms an \((n–1)\)-dimensional subspace of \( \mathbb{C}^n \). With respect to an appropriate basis, such an \( L \) can be represented by a diagonal matrix of \((n–1)\) 1’s and a root of unity \( \neq 1 \). A finite unitary reflection group is a finite group of unitary transformations that is generated by the reflections it contains. Hilbert: To each finite unitary group \( G \) on \( \mathbb{C}^n \) corresponds a finite set of homogeneous \( G \)-invariant polynomials \( p_1, \ldots, p_N \) such that every \( G \)-invariant polynomial \( f \) can be expressed in the form \( f(z) = g(p_1(z), \ldots, p_N(z)), \forall z \in \mathbb{C}^n \), where \( g \) is some polynomial on \( \mathbb{C}^n \). Chevalley [76, this Bib] added that one can take \( N = n \) if \( G \) is a reflection group; Shephard & Todd [386, this Bib] showed \( N > n \) except when \( G \) is a reflection group.


From Meisters’ Bibliography on Polynomial Maps


[206] Engelbert Hubbers. *Cubic Similarity in Dimension Five.* Doctors thesis, University of Nijmegen, Nijmegen, The Netherlands, July 1996. An incredible and beautiful piece of work. Hubbers gives in this 50-page paper a complete classification of all Drużkowski cubic-linear maps $F := X + (\text{diag}[AX])^2AX$ of $k^2$ into itself with nilpotent Jacobian $3(\text{diag}[AX])^2A$ of the homogeneous part $(\text{diag}[AX])^2AX$, where $k$ is any algebraically closed field. He then uses this classification to find all representatives of the cubic-similarity relation in dimension five. This is an important and difficult step in the program outlined by Meisters at the October 1992 Luminy Conference on Polynomial Maps.

[207] Engelbert-M. G. M. Hubbers. The Jacobian Conjecture: Cubic Homogeneous Maps in Dimension Four. Master’s thesis, Katholieke Universiteit Nijmegen, Department of Mathematics and Computer Science, Nijmegen, The Netherlands, February 17, 1994. This thesis contains an excellent summary of results on the Jacobian Conjecture and presents many interesting and useful examples of polynomial maps with various properties. Chapter 1: A short historical summary including a proof that the general Jacobian Conjecture reduces to the case where $F = I − H$ with $H$ cubic-homogeneous; and David Wright’s result that this case holds for $n = 3$. Chapter 2: Hubbers generalizes Wright’s result to $n = 4$ by means of a complete classification of all cubic-homogeneous polynomial maps in dimension 4 when $\det F'(x) = 1$; all these $F$ turn out to be injective! Chapter 3: Proof that all cubic-homogeneous maps in two dimensions with coefficients in a uniform factorization domain can be written in a very simple way. Chapter 4 takes up the four-dimensional case again, but for those special $F = I − H$ where $H$ is in Drużkowski’s cubic-linear form. (Drużkowski has shown that to prove the Jacobian Conjecture it suffices to prove it, but in all dimensions, for maps of this special type.) Hubbers gives a complete classification of cubic-linear maps in four dimensions; and uses this to prove Meisters’ conjecture about the cubic-similarity representatives in dimension four. Hubbers then proves that the Jacobian Conjecture is true for cubic-linear maps in dimensions $n \leq 7$. Chapter 5 discusses locally nilpotent derivations; and gives examples of quadratic-homogeneous Jacobian matrices in dimension four that are not strongly nilpotent. Finally, in Chapter 6 Hubbers begins to investigate these matters in dimension five. Now see his newer 50-page paper (his Doctor’s Thesis?) [206].

[208] Zbigniew Jelonek. The set of points at which a polynomial mapping is not proper. to appear, (Year?).


[215] R. E. Kalman. See Barabanov: According to the Math Review of Barabanov’s paper, “By strengthening the assumptions in Aizerman’s Problem, R. E. Kalman conjectured that if $\varphi(\sigma) \in (\alpha, \beta) \forall \sigma$, then the origin is globally asymptotically stable”.


M. Kirezci. The Jacobian Conjecture I. *Bull. Tech. Univ. Istanbul*, **43**:421–436, (1990). **In brief:** This paper gives a formal inverse $G = (G_1, \ldots, G_n)$ in the ring $k[[X]]$ of formal power series in $X = (X_1, \ldots, X_n)$ for mappings of the form $F = X - H$ where each component $H_i$ is a cubic homogeneous polynomial and the jacobian matrix $J(H)$ is nilpotent; and a recursion formula for the homogeneous parts of the components of $G$.


M. Kwieciński et al. See van den Essen.


From Meisters’ Bibliography on Polynomial Maps


Note that for quadratic maps $g$ of $k^n$ into itself, we have $g(x) - g(y) = g'(x+y)(x-y) ;$ so $det g'(x) \neq 0$ implies that $g$ is $1-1$ (injective).


[247] R. Daniel Mauldin, Editor. The Scottish Book: Mathematics from the Scottish Café. Scottish Book Conference, North Texas State University, May 1979. Birkhäuser, Boston-Basel-Stuttgart, First U. S. edition, 1981. “Problem 79” (Mazur, Orlicz): A POLYNOMIAL $y = U(x)$ maps, in a one-to-one fashion, a space $X$ of type (B) onto a space $Y$ of type (B); the inverse of this mapping $x = U^{-1}(y)$ is also polynomial. Is the polynomial $y = U(x)$ of first degree? Not decided even in the case when $X$ and $Y$ are a Euclidean plane.” An “Addendum” states “Trivial” and gives the triangular example in $n$ dimensions. CURIOUS! Did they hear about Keller’s Conjecture from someone, but somehow get it wrong? They have taken the conclusions as the hypotheses, and lost Keller’s hypothesis that $det U'(x)$ be a nonzero constant! It is Keller’s Question that is not decided even in two dimensions; nonlinear polynomials exist in all dimensions $\geq 2$, but have been classified only in dimension two.


[262] G. H. Meisters. The Markus-Yamabe Conjecture Implies the Keller Jacobian Conjecture. In Massimo Furi, editor, *Proceedings of the International Meeting on Ordinary Differential Equations and their Applications, at Firenze, Italy, to celebrate the 70th Birthdays of Roberto Conti and Gaetano Villari, IMODEA*, September 20 1993. A counterexample to the Keller Jacobian Conjecture in some dimension allows one to construct also a polynomial vector field counterexample to the Markus-Yamabe Conjecture on Global Asymptotic Stability in some larger dimension. This has been known for several years by several people including the author, Czeslaw Olech, Gilles Fournier, and Mario Martelli.


July 4–8, 1994, Conference on Invertible Polynomial Maps held at Curaçao, The Netherlands Antilles. This paper shows by examples and calculations that many, even if not all, dilations of polymorphisms are conjugate to (the same dilation of) their linear part by means of another polymorphism. Thus various questions arise as to which polymorphisms are dilation-conjugate to their dilated linear part by polymorphisms, holomorphisms, diffeomorphisms, or at least homeomorphisms. In particular, the cubic-linear linearization conjecture, stated in the last sentence of the penultimate paragraph on page 85, has been settled by van den Essen [146] and Gorni & Zampieri [192]. The easier cubic-homogeneous case was settled by a counterexample found by van den Essen soon enough after the Curaçao Conference to be included at the end of the conference proceedings [139] as item [138]. This cubic-homogeneous counterexample first appeared as a Nijmegen Report [136], and Meisters paid van den Essen $100, as publicly promised at the Curaçao Conference, for this cubic-homogeneous counterexample. But the cubic-linear linearization counterexample was harder to obtain.


From Meisters’ Bibliography on Polynomial Maps

21


[307] A. Nowicki. Rings and fields of constants for derivations in characteristic zero. *Please tell me the Journal, vol? (number?):pages?, month? (year?)*. Brian Coomes will be interested to read this paper. Please send reprints to him and Meisters.


[329] T. Parthasarathy et al. See van den Essen, Olech, and Ravindran.


[338] R. Peretz. On the Real Jacobian Conjecture in Two Dimensions. In Marco Sabatini, editor, *Recent Results on the Global Asymptotic Stability Jacobian Conjecture*, Università di Trento, I–38050 POVO (TN) ITALY, September 14–17 1993. Dipartimento di Matematica, Università di Trento, Italia. The fifth lecture on Tuesday, September 14, 1993. Very interesting even though he could not complete his argument in the time (one hour) allotted. We anxiously await his manuscript to see if he has indeed proved it! Alas! Ronen joins the elite club of those who have produced “proofs” of the 2-dimensional Jacobian Conjecture, only to find later that there is a flaw in the argument. Many good mathematicians have done this before him; and many of these arguments nevertheless contain interesting and useful ideas. Fortunately, Ronen has rewritten his paper, to bring out these valuable parts.


[345] R. Plastock. Homeomorphisms between banach spaces. Transactions of the Amer. Math. Soc., 200:169–183, December (1974). $\int_0^\infty \sup_{t \geq s} \|F'(x)\|^{-1} = \infty \Rightarrow \ldots$


[353] V. M. Popov. What is the reference for Popov’s paper(s) on Aizerman’s Problem? See Barabanov, Fitts, Pliss, and Yakubovich.


[372] W. Rudin. Proper Holomorphic Maps and Finte Reflection Groups. *Indiana Univ. Math. J.*, 31(5), September/October (1982). Motivated by Alexander’s Theorem [27, this Bib], Rudin investigates what can be said about proper holomorphic maps of the open unit ball $B$ of $\mathbb{C}^n$ into other regions (connected open subsets) of $\mathbb{C}^n$.


[374] L. Rudolph. Embeddings of the line in the plane. *J. Reine Angew. Math.*, 337:113–118, (1982). A knot-theory proof of the Embedding Theorem of Abhyankar-Moh: Let $(p, q)$ be an embedding of the line in the plane. That is, let $(p(t), q(t))$ be a pair of polynomials in $k[t]$, such that $(p, q) : k^1 \to k^2$ is one-to-one and the tangent vector $(p'(t), q'(t))$ is never $(0, 0)$. Then there exist polymorphisms $a$ of $k^1$ and $A$ of $k^2$ so that $A(p(a(t)), q(a(t))) = (t, 0)$.


[378] M. Sabatini. Global Asymptotic Stability of Critical Points in the Plane. *Rend. Sem. Mat. Univers. Politecn. Torino; Dynamical Systems and O. D. E.*, 48(2):97–103, (1990). A function $h : \mathbb{R}^2 \to \mathbb{R}$ is said to have the property $(\mathcal{H})$ if there exists a positive integer $N$ such that, for any real number $\lambda$, the number of connected components of $h^{-1}(\lambda)$ is not greater than $N$. **Theorem:** If one of the components of $f$ has the property $(\mathcal{H})$, and if $\det f'(x) > 0$ and $\text{trace} f'(x) < 0 \forall x \in \mathbb{R}^2$, then $f$ is injective.


[384] A. Samuelsson. A local mean value theorem for analytic functions. *Amer. Math. Monthly*, 80:45–46, January (1973). **Theorem:** If $f$ is analytic at $z_0$, then there is a neighborhood $\mathcal{N}$ of $z_0$ such that, for each $z_1 \in \mathcal{N}$, there is a point $z$ satisfying

$$| z - \frac{1}{2}(z_0 + z_1) | < \frac{1}{2} | z_1 - z_0 |,$$

such that $f(z_1) - f(z_0) = (z_1 - z_0)f'(z)$. University of Göteborg.


[387] V. Shpilrain and J.-T. Yu. Polynomial retracts and the Jacobian Conjecture. Preprint, 1996. Proposes a new approach to attack the two-dimensional Jacobian Conjecture via the concept of a retract of a polynomial ring introduced in [91]. The authors conjecture that if $p \in k[X,Y]$ has a unimodular gradient (i.e., $1 \in$ belongs to the ideal in $k[X,Y]$ generated by the partial derivatives of $p$), then $k[p]$ is a retract; and show that this would imply the Jacobian Conjecture. But see van den Essen [143] for counterexamples to their conjectures.


[390] M. K. Smith. Stably Tame Automorphisms. *J. Pure and Applied Algebra*, 58:209–212, (1989). **MR 90f:13005.** Defines **linear, triangular, tame,** and **stably tame** polynomial automorphisms; and **locally nilpotent** derivations. Shows that a previously unpublished, not-known-to-be-tame, 4-dimensional cubic-homogeneous example of David Anick is stably tame (has a tame 5-dimensional extension). Also shows that the, not-known-to-be-tame, 3-dimensional Bass-Nagata example is stably tame. David Wright [unpublished] has also shown that the Bass-Nagata automorphism is stably tame.


D. M. Snow. Complex Orbits of Solvable Groups. *Proceedings of the Amer. Math. Soc.*, **110**(3):689–696, November (1990). Proves Structure Theorems such as: An orbit of a real solvable Lie group in projective space that is a complex submanifold is isomorphic to $\mathbb{C}^k \times (\mathbb{C}^*)^m \times \Omega$, where $\Omega$ is an open orbit of a real solvable Lie group in a projective rational variety.


M. Spivak. *A Comprehensive Introduction to Differential Geometry*, volume Four. Publish or Perish, Inc., 6 Beacon Street, Boston, Mass. 02108, USA, first isbn 0-914098-03-9 edition, 1975. Jörgens Theorem (pages 165–170): If $\phi: \mathbb{R}^2 \to \mathbb{R}$ is a function on the whole plane whose Hessian $H(\phi) = \phi_{xx}\phi_{yy}(\phi_{xy})^2 = 1$ (or any positive constant), then $\phi$ is a quadratic polynomial in $x$ and $y$.


Halszka Tutaj-Gasińska. A Note on the Solution of the Two-Dimensional Ważewski Equation. *Bull. Polish Acad. Sci. Math.*, **44**(2):245–249, (1996). If the solution \( x(t,x_0) \) of the two-dimensional Ważewski Equation \( \dot{x} = [F'(x)]^{-1}a \), where \( x(0,x_0) = x_0 \) and \( a \) are in \( \mathbb{R}^2 \), is a *polyflow* (polynomial in the initial-condition parameters which are the components of the vector \( x_0 \)), then it is also polynomial in \( t \). (This is interesting because only one of the six canonical forms for 2-dimensional polyflows is polynomial in \( t \) as well as in \( x_0 \): namely, the canonical form \( x = (u_0, v_0 + \varphi(u_0)t) \), where \( \varphi \) is a polynomial of degree \( \geq 1 \).) See [45], [256], [273], and [405].

Vasilić Ivanović Vasyunin. Counterexamples to strong nilpotence in dimension five. Personal Communication, June 20, 1989. While at the STEFAN BANACH INTERNATIONAL MATHEMATICAL CENTER, 25 Mokotowska ul., Warszawa, Poland, the Russian mathematician Vasilić Ivanović Vasyunin (B. I. Vasconian), from L. O. M. I. Steklov Institute [Leningrad, Russia] gave me [Meisters] some 5-dimensional counterexamples to the following question which I had posed to him three weeks earlier on May 30, 1989. QUESTION: Do the three hypotheses (1) \( J : \mathbb{R}^n \to M_n(\mathbb{R}) \) is linear, (2) \( J(x)y \equiv J(y)x \), and (3) \( J(x)^n \equiv 0 \), imply the stronger nilpotence condition that each \( n \)-factor product \( J(a) \cdot \cdot \cdot J(z) \) is zero (where \( a, \ldots, z \in \mathbb{R}^n \) ? Vasyunin’s answer: “Not in dimension five”. I received four more five-dimensional examples from Wasia on February 12, 1990; and nine additional examples in a letter from Stockholm dated December 1, 1990. All of these examples, along with some others, have been included in my paper with Czesław Olech: Strong Nilpotence Holds in Dimension up to Five Only, *Linear and Multilinear Algebra*, **30**(1991), 231–255.


S. Walcher. On Sums of Vector Fields. *T*EX Preprint, September 1995. Discusses one case where the integration of a sum of vector fields is reducible to the integration of the summands; with applications to stably tame group actions and mathematical biology.


T. Ważewski. Sur un problème de caractère intégral relatif à l’équation \( \frac{\partial x}{\partial y} + Q(x, y)\frac{\partial y}{\partial y} = 0 \). *Mathematica, Cluj-Napoca, Romania*, **8**:103–116, (1934). Receve le Mars 1932.


Jan C. Willems. What is the reference for Willems’s paper(s) on the aizerman problem? See Brockett. According to Jürgen Moser (in a letter to me dated 13 April, 1983), “The Problem of Aizerman has been solved, by a counterexample; see V. A. Pliss and J. C. Willems”.


D. Wright. The amalgamated free product structure of (the group) \( GL_2(k[X_1, \ldots, X_n]) \) and the weak Jacobian Theorem for two variables. *J. Pure Appl. Algebra*, **12**:235–251, (1978).


[431] F. Xavier. Invertibility of Bass-Connell-Wright polynomial maps. *Math. Ann.*, 295:163–166, (1993). Theorem: Let $F(x) = I - H(x)$ be a polynomial map of $C^n$ into itself with $H(x)$ homogeneous, $[H'(x)]^k \equiv 0$, and $[H'(x)]^{k-1} \not\equiv 0$. If $k \geq 3$ and $\cup \{ \ker[H'(x)]^{k-1} : H(x) = 0, x \neq 0 \} \neq C^n$, then $F$ is invertible.


[436] V. A. Yakubovich. What is the reference for Yakubovich’s paper(s) on Aizerman’s Problem (in the 1960’s)? See Math Reviews MR 38#4172 and MR 35#4036.


J.-T. Yu. On the Jacobian Conjecture for Global Asymptotic Stability. *Journal of Differential Equations*, 104:11–19, (1993). *MR 94e:26022*. By introducing the variation \( \dot{x} = -f'(x)^{-1}f(x) \) on Ważewski’s Equation \( \dot{x} = f'(x)^{-1}v \) Zampieri obtains some interesting results. Others have then been able to get even further interesting results from this idea. E.g., see [89].


G. Zampieri. Finding Domains of Invertibility for Smooth Functions by Means of Attraction Basins. *Journal of Differential Equations*, 104:11–19, (1993). *MR 94e:26022*. By introducing the variation \( \dot{x} = -f'(x)^{-1}f(x) \) on Ważewski’s Equation \( \dot{x} = f'(x)^{-1}v \) Zampieri obtains some interesting results. Others have then been able to get even further interesting results from this idea. E.g., see [89].


G. Zampieri and G. Gorni. On the Jacobian Conjecture for Global Asymptotic Stability. *Journal of Dynamics and Differential Equations*, 4(1):43–55, January (1992). Their strategy to tackle the injectivity of \( f \), based on an auxiliary boundary value problem, is shown to be successful if the norm of the matrix \( I + J(x)^{T}J(x)/\text{det}J(x) \) is bounded, or at least grows slowly (for instance, linearly) as \(|x| \to +\infty\).


[463] V. Zurkowski. Polynomial Flows in the Plane: A Classification Based on Spectra of Derivations. *Journal of Differential Equations*, 120:1–29, (1995). This work was partly done while Zurkowski was a postdoctoral member of the Institute for Mathematics and its Applications at the University of Minnesota, Minneapolis, Minnesota; and a Gibbs Instructor at Yale University, New Haven, Connecticut. It is important because its methods are not *a priori* restricted to two dimensions, as are those used in the paper on the same subject by H. Bass & G. H. Meisters: Therefore there is some hope that some progress could be made on the, yet to be done, classification of polyflows in *three* dimensions. To this end, someone should now compare the three papers on the classification of two-dimensional polyflows: Suzuki [1977], Bass & Meisters [1980–85], and this one by Zurkowski [1990–95].
Part II. Addresses of Authors

Abate, M.
Abhyankar, Shreeram Shankar. ⟨ram@cs.purdue.edu⟩. Div Math Sci, Purdue U, West Lafayette, IN 47907 USA.

Adjamagbo, Kossivi. “Pascal”. ⟨adja@ccr.jussieu.fr⟩. Mathématiques, Université Paris VI, 4 place Jussieu, 75252 Paris Cedex 05, France.

Ahern, Patrick R. ⟨ ⟩. Dept Math, U Wisconsin, Madison, WI 53706 USA.

Alev, Jacques. ⟨jle@ccr.jussieu.fr⟩. Université Paris VI, 4 place Jussieu, 75252 Paris Cedex 05, France.

Alexander, Herbert J. ⟨u22330@uicvm.uic.edu⟩. U Ill Chicago, M/C 249, Box 4348, Chicago, IL 60680–4348. Tel.: (office) (312) 413–2158; (home) (708) 256–6596

Alexandrov, Victor A. ⟨alex@math.nsk.su⟩. Math. Inst., Novosibirsk-90, 630090, Novosibirsk, Russia.

Alg-geom@publications.math.duke.edu

Andersén, E.

Angemüller, G.

Anick, David Jay. ⟨dja@bourbaki.mit.edu⟩. Dept Math, MIT, Cambridge, MA 02139, USA.

Appelgate, Harry. City College of New York, NY 10031, USA

Arnold, Vladimir Igorevič. Moskovskii U, Mehmat, Moscow 117234, Russia.

Ax, James. 1427 Chautauqua Blvd, Pacific Palisades, CA 90272 USA.


Banach Center (Warszawa). ⟨banach@impan.impan.gov.pl⟩. 25 Mokotowska ulica, Warszawa, Poland.


Bebernes, Jerry. ⟨bebernes@newton.colorado.edu⟩. Applied Mathematics, Campus Box 426, University of Colorado, Boulder, CO 80309–0001.

Beckner, Michael.

Bedford, E.

Berna, Josep. ⟨IMAT0@cc.uab.es⟩. Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193-Bellaterra, Barcelona, Spain.

Białynicki-Birula, Andrzej. Dept Mathematics, U Warszawa, Warszawa, Poland


Browder, Felix E.

Buchberger, Bruno.
Byrne, Catriona. ⟨sv-byrne@dcfrz1.das.net⟩. Math Editor, Springer-Verlag, (Lecture Notes Series) Tiergartenstrae 17, 69121 Heidelberg, Postfach 10 52 80, D-69042 Heidelberg, Germany.

Campbell, Laughlin Andrew. ⟨campbell@aerospace.aero.org⟩. M 1102 Aerospace Corp., P. O. Box 92957, Los Angeles, CA 90009.

Cassou-Nogues, Pierrette (pierrette@pucc.princeton.edu). Centre de Recherche en Math. de Bordeaux, U de Bordeaux I, 33405 Talence cedex, France.

Chadzyński, Z.

Charzyński, J.

Chau, Nguyen Van. ⟨ ⟩. Institute of Mathematics Hanoi, Viên Toán Hoc Inst. Math., P. O. Box 631 BO HO, 10,000 Hanoi, Vietnam. Tel.: 43303. Telex 411525 NCSR VT.

Cheng, Charles Ching-an. ⟨cheng@vela.acs.oakland.edu⟩. Oakland U Rochester, Michigan.

Chicone, Carmen. ⟨carmen@chicone.math.missouri.edu⟩. Math Sci Bldg, U Missouri, Columbia, MO 65211.

Cima, Anna. ⟨cima@ma2.upc.es⟩. Departament de Matemàtica Aplicada II, E.T.S. d'Enginyers Industrials de Terrassa, Universitat Politècnica de Catalunya, Colom 11, 08222 Terrassa, Barcelona, Spain.

C.I.R.M. ⟨azm@cirm.univ-mrs.fr⟩. Luminy, Marseilles, France.

Connell, Edwin. Dept Math and Comp Sci, U Miami, P. O. Box 249085, Coral Gables, Florida, FL 33124.

Coomes, Brian Arthur. ⟨coomes@math.miami.edu⟩. Dept Math and Comp Sci, U Miami, P. O. Box 249085, Coral Gables, Florida, FL 33124.

Daigle, Daniel. ⟨daniel@mathstat.uottawa.ca⟩. Daniel Daigle & Anne-Marie Rajotte, 188 Maybury, Hull, Québec, Canada J9A 2A8.

Dean, Carolyn. ⟨cdean@math.lsa.umich.edu⟩. U Michigan, Ann Arbor.

De Marco, Giuseppe. ⟨gdemarco@pdmat1.unipd.it⟩. Dip. di Matematica Pura ed Applicata, Via Belzoni 7, I-35131 Padova, Italy. Tel.: ++39(49)831932.

Deng, Bo. ⟨bdeng@ode.unl.edu⟩ & ⟨bdeng@unlinfo.unl.edu⟩. Dept Math & Stat, U Nebraska, Lincoln, NE 68588 - 0323, U.S.A.

Derksen, Harm G. J. ⟨hderksen@math.unibas.ch⟩. U Nijmegen, Toernooiveld, 6525 ED Nijmegen, Netherlands.

Deveney, James K. ⟨jdeveney@cabell.vcu.edu⟩. Virginia Commonwealth U Richmond.

Dieudonné, Jean A.

Dillen, Franki.

Dixon, P. G.

Drużkowski, Ludwik M. ⟨druzkows@im.uj.edu.pl⟩. Instytut Matematyki, Uniwersytet Jagiellonski, ul. Reymonta 4, 30–059 Kraków, Poland. Tel.: 48(12)336377–585.

Eggermont, Christian.

Engel, W.

Essen, Arno R. P. van den. ⟨essen@sci.kun.nl⟩. (Sandra & Raissa) Dept Math, U Nijmegen, Toernooiveld 6525 ED Nijmegen, The Netherlands.

Esterle, J.

Feffer, Robert. ⟨fessler@math.unibas.ch⟩. Basel, Switzerland. Feffer is a 1991 student of K. P. Rybakowski at U Freiburg, Germany. (Rybakowski has since moved to Trieste.) Feffer had a position in the Dept. of Computer Science (Inst. f. theoretische Informatik) at ETH-Zentrum, CH-8092 Zürich, Switzerland. Now in Math Dept at Basel, Switzerland. But lives in Freiburg, Germany (about 60 km to the north).

Finston, David R. ⟨dfinston@nmsu.edu⟩. Las Cruces, New Mexico.

Formanek, Edward.
Forstneric, Franc. (forstner@math.wisc.edu) & (franc.forstneric@uni-lj.ac.mail.yu).
U Wisconsin, Madison, WI-53706, USA, Tel.: (office) 1 - (608) - 263 - 4880.
Freudenburg, Gene. (gfreuden@math.bsu.edu)? U So Indiana, Evansville.

Friedland, Shmuel.

Gale, David.

Gasull, Armengol. (GASULL@mat.uab.es) or (imgasull@cc.uab.es) Departament de Matemàtiques, Edifici II, Universitat Autònoma de Barcelona, 08193-Bellaterra, Barcelona, Spain.

Gordon, William B.

Gorni, Gianluca. (gorni@udmi5400.cineca.it). Università di Udine, Dip. di Matematica e Informatica, V. Zanon, 6, 33100 Udine, Italy. Tel.: ++39(0432)272225.

Greenig, Doughlas. (greenig@math.berkeley.edu). Student of Pugh. **Title** of thesis? **Year** of Ph. D degree? (From Berkeley?) Where is he now? New e-mail address?

Guralnick, Robert M. (guralnic@mtha.usc.edu). U Southern Calif.

Gerstenhaber, Murray.

Ghutsuk, Aleksei A. Moscow State University, department of Mathematics, Vorobyovi Gori, 117234, Moscow, Russia.

Gutierrez, Carlos. (gutp@impa.br). Inst. de Matematica Pura e Aplicada (IMPA), Estrada Dona Castorina, 110, Río de Janeiro, R. J. 22460, Brazil.

Hadamard, Jacques.

Halanay, Aristide.

Hamann, Eloise Ann. (hamann@mathcs.sjsu.edu). San José State U, California.

Heitman, Raymond C. ( ).

Honsbeek, Mascha.

Huang, Xiaogang.

Hubbers, Engelbert. (hubbers@sci.kun.nl). Dept Math, U Nijmegen, Toernooiveld 6525 ED Nijmegen, The Netherlands.

Jagžev, A. V. [See Yagzhev, A. V.]

Jelonek, Zbigniew.

Kaliman, Shulim I.

Kalman, Rudolf E.


Kestelman, H.

Kirezi, Murat.

Kishimoto, Kazuo.

Kraft, Hanspeter. (kraft@math.unibas.ch).

Mathematisches Institut, Universität Basel, Rheinsprung 21, CH-4051 Basel, Suisse (Switzerland).

Krasinski, Tadeusz.

Kulk, Wouter van der.

Kwieciński, Michał. (umkwieci@im.uj.edu.pl)? Jagellonian U, ul. Reymonta 4, PL-30–059, Kraków, Poland.

Laffey, Thomas J.

Lang, Jeffrey.
Li, Bang-He. ⟨Libh@iss06.iss.ac.cn⟩. Institute of Systems Science, Academia Sinica, Beijing 100080, P. R. China.

Li, Wei. ⟨wei@gauss.math.mcgill.ca⟩. McGill U, Montreal, Quebec, Canada.

Llibre, Jaume. ⟨IMAT0@cc.uab.es⟩. Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193-Bellaterra, Barcelona, Spain.

Mackey, Milon. ⟨mackey@cello.hpl.hp.com⟩. Apparment “G”, 765 Live Oak Ave., Menlo Park, CA 94025.

Magnus, Arne.

Mañosas, Francesc. ⟨manyosas@manwe.mat.uab.es⟩. Departament de Matemàtiques, Edifici II, Universitat Autònoma de Barcelona, 08193-Bellaterra, Barcelona, Spain.

Markus, Lawrence.

Mathematica Special Interest Group. ⟨mathgroup-request@yoda.ncsa.uiuc.edu⟩.

McKay, James H. ⟨mckay@vela.acs.oakland.edu⟩.

McLeod, Robert M. ⟨mcleod@vax001.kenyon.edu⟩. Kenyon College, Gambier, Ohio.

Milnor, John. ⟨jack@math.sunysb.edu⟩.

Inst for Math Sci, SUNY at Stony Brook, Math Bldg, Stony Brook, NY 11794–3660.

Mitchell, J.

Miyanishi, Masayoshi.

Moh, Tzuong Tsieng. Purdue University.

Moser, Jürgen.

Nagata, Masayoshi.


Nelson, Edward. ⟨nelson@math.princeton.edu⟩. Princeton University, New Jersey.

Newman, Donald J.

Niitsuma, Hiroshi.

Nijenhuis, A.

Nikaidô, Hukukane.

Nousiainen, Pekka.

Nowicki, Andrzej. ⟨anow@mat.uni.torun.pl⟩. Institute of Mathematics, N. Copernicus University, ul. Chopina 12/18, 87-100 Toruń, Poland.

Oda, Susumu.

Olech, Czesław. ⟨olech@impan.gov.pl⟩. ul. Nowy Świat 23/25 m.1, 00–029 Warszawa, Poland.

Onishi, Hironori.

Palais, Richard S.

Panchal, Champak D.

Parthasarathy, Thiruvengada. ⟨tps@isid.ernet.in⟩. Indian Statistical Institute, 7, S. J. S. Sansav Marg, New Delhi 110 016, India. Tel.: ++91(11)6868114.

Pauer, Franz.

Peretz, Ronen. ⟨ronenp@black.bgu.ac.il⟩. Ben Gurion U of the Negev, Faculty of Natural Sciences, Dept. of Mathematics & Comp. Science, P.O. Box 653, Beer-Sheva 84105, Israel.

Pfeifhofer, Marlene.

Pinčuk, Serguei I. ⟨⟩. Dept Math, Bashkir State University, Ufa, 450074, Russia.

Pittaluga, Marilena.
Pliss, Victor A.
Ploski, Arkadiusz.
Polynomial-Solving Group. (general-posso@dm.unipi.it). Italy.
Pugh, Charles. (pugh@math.berkeley.edu). Berkeley, California.
Rabier, Patrick J. ( ). Dept Math, U Pittsburgh, Pittsburgh, Pennsylvania, PA 15260, USA.
Randall, John D.
Ray, William O.
Rejai, Behshad. (rejaib@ roses.rockwell.com). Rockwell-International, Artificial Intelligence, 1811 California Street, Huntington Beach, CA 92648.
Rentschler, Rudolf.
Richardson, R. W. Jr.
Robbiano, Lorenzo.
Rosay, Jean-Pierre.
Rosenholtz, Ira.
Rosenlicht, Maxwell.
Rubel, L.
Rudin, Walter. Professor Emeritus, Mathematics Dept, 805 Van Vleck Hall, U Wisconsin, 480 Lincoln Drive, Madison, WI 53706. Home Tel.: (608) 231–3248
Rusek, Kamil. (rusek@im.uj.edu.pl). Jagiellonian University, ul. Reymonta 4, PL-30–059, Kraków, Poland.
Sabatini, Marco. (sabatini@science.unitn.it) or (sabatini@itnvax.science.unitn.it).
Dipartimento di Matematica, Università Delgli Studi di Trento, I-38050 Povo (Trento) Italy. Tel.: ++39 (461) 881 670. Preprints are numbered UTM number.
Samuelsson, Åke.
Schwarz, Gerald.
Serre, Jean-Pierre. Collège de France, Chaire d’Algèbre et Géométrie, F - 75231 Paris Cedex 05.
Shpilrain, Vladimir (shpi@ math.ucsb.edu). Dept Mathematics, U Calif Santa Barbara, CA 93106.
Siegel, Carl Ludwig.
Simon, Carl P. Dept Mathematics and Economics, U Michigan, Ann Arbor, MI 48109.
Skibiński, P.
Smith, Martha K. (mks@math.utexas.edu) or (mks@ fireant.ma.utexas.edu).
Dept Math, U Texas, Austin, Texas, TX 78712.
Snow, Dennis M. ( ). Dept Mathematics, U Notre Dame, Notre Dame, IN 46556.
Sotomayor-Tello, Jorge Manuel. (sot@ime. nsp.br or sot@impa.br). IMPA: Instituto de Matemática Pura e Aplicada, Edificio Lelio Gama, Estrada Dona Castorina, 110, CEP 22460,–Jardim Botânico, Rio de Janeiro, Brazil.
Spivak, Michael.
Stein, Yosef. (yosef.st%ilctehol@vms.huji.ac.il). Tel Aviv U, Holon, Israël.
Sternberg, Shlomo.
Strang, Gilbert (“Gil”). ⟨gs@bourbaki.mit.edu⟩. M I T, Cambridge, Massachusetts.

Sturmfels, Bernd.

Swan, R. G.

Sweedler, Moss E.

Taussky, Olga.

Trotman, David. ⟨trotman@gyptis.uni-mrs.fr⟩. Mathématiques, U F. R. de Université de Provence; 3, pl. Victor Hugo; F - 13331 - Marseille; Cedex 03 France.

Tutaj-Gasińska, Halszka K. ⟨htutaj@im.uj.edu.pl⟩. Kraków, Poland.

Vasyunin, Vasilii Ivanovič. (V. I. Vassyunin.) e-mail: ( ). L. O. M. I. Steklov Institut, 27 Fontanka Street, Saint Petersburg (Leningrad) 191011, Russia. Gary met him at the Banach Center, 25 Mokotowska ulica, Warszawa, Poland, on May 30, 1989. Polish nickname “Wasia”.

Vidossich, Giovanni.

Vitushkin, Anatolii Georgievich. MIAN, ul. Vavilova 42, Moscow, GSP-1, 117966, Rusia. I discussed the Jacobian Conjecture with this blind mathematician at Olech’s flat, during the 1982-Warszawa ICM which was held in August 1983.

Walcher, Sebastian. ⟨swalcher@NMSU.edu⟩. Mathematisches Institut TU München, 80290 München, Germany.

Wang, Stuart Sui-Sheng. ⟨swang@vela.acs.oakland.edu⟩. Math Dept, Oakland U, Rochester, MI 48309.

Ważewski, Tadeusz. [Wrzesień 24, 1896 – Wrzesień 5, 1972] (Wrzesień = September). He was born in Wygnanka, a village near Tarnów, Poland. After graduating from Tarnów High School in 1914 he enrolled in the Jagiellonian University in Kraków where he studied physics and mathematics; then spent the years 1921–1923 as a graduate student at the University of Paris where he received his doctor’s degree in 1924 with a thesis about connected continua not containing simple closed curves: Sur les courbes de Jordan ne renfermant aucune courbe simple fermée de Jordan Ann. Soc. Polon. Math. 2 (1923) 49–170. His doctoral examination committee consisted of Emil Borel, Arno Denjoy, and Paul Montel. He received his second (habilitation) degree from the Jagiellonian University in 1927 on the basis of a paper on rectifiable continua in connection with absolutely continuous functions and mappings. In 1933 he was appointed an extraordinary professor at the Jagiellonian University. On November (Listopad) 6, 1939, Tadeusz Ważewski, along with a large group of university professors of various schools in Kraków, was arrested by the Nazi occupation forces in Poland and deported to a concentration camp. He was released before the end of the war and spent the rest of the German occupation in Kraków teaching in the underground university and continuing his research activity. He was promoted to ordinary professor (full professor) in 1945. When the Polish Academy of Sciences was founded in 1952, he became its corresponding member and was elected a full member in 1957. Ważewski was one of the founders of the Mathematical Institute of the Polish Academy of Sciences and was head of its Kraków branch for many years. He was President of the Polish Mathematical Society for 1957–59; and a long time editor of the journal Annales Polonici Mathematici. The Ważewski Equation [420] \(dx/dt = F'(x)^{-1}a\), for \(x\) and vector parameter \(a\) both in \(\mathbb{R}^n\), plays an important role in several papers in this Bibliography: E.g., in [273], [257], [453], and [89]. He has long been famous for a powerful topological-geometric method that he introduced, known as “The Ważewski Method”, for establishing the existence of certain types of solutions of ordinary differential equations. Reference: TADEUSZ WAŻEWSKI, SELECTED PAPERS, PWN-Polish Scientific Publishers, Warszawa 1990, isbn 83-01-09733-7.

Winiarski, Tadeusz.

Willems, Jan C.

Wright, David. ⟨wright@einstein.wustl.edu⟩. Mathematics Dept, Washington U, St. Louis, MO 63130.

Wu, Xiaolong.

Xavier, Frederico. Dept Mathematics, U Notre Dame, P. O. Box 398, Notre Dame, IN 46556. Tel.: (219) 631–6288. Fax: (219) 631–6579.

Xu, Yansong. E-mail? Address? Please send reprints to Meisters.
Thanks to Marc Chamberland (marc@daniel.math.mcmaster.ca) (M&S, McMaster U, Hamilton, Ontario, Canada, L8S 4K1; Tel.: (905) 525-9140 x27589) for information about Xu’s Ph.D. thesis. New Address: Dept Mathematics & Computer Science, Grinnell College, IA 50112, USA

Yagzhev, A. V. [Jagˇ zev] Khabarovsky Polytechnic Institute, Khabarovsky, Russia.
(please send meisters Yagzhev’s current address.)


Yu, Jie-Tai. ⟨yujt@hkuxa.hku.hk⟩. August 1995: Dept Mathematics, University of Hong Kong, Hong Kong. 1993 Ph.D. thesis, Univ. of Notre Dame. Organizer of a conference on Algebra and Geometry, June 10–14, 1996 at the University of Hong Kong. Expect 20 speakers from all over the world and about 50 participants.

Zampieri, Gaetano. ⟨Zampieri@dm.unito.it⟩. Dipartimento di Matematica Università di Torino, via Carlo Alberto 10, 10123 TORINO, Italy.

Zariski, Oscar [1899–1986]. Born “Ascher Zaritsky”, April 24, 1899, in Kobrin (Kobryń), a small Polish-Russian-Jewish town (only 140 miles east of Warszawa) in “The Jewish Pale of Settlement” in Belorussia, which borders on Poland, Lithuania & Latvia. (Poland has been partitioned repeatedly by her neighbors—Russia, Austria, and Germany; at the time of Zariski’s childhood, this region was ruled by Russia.) He was the sixth child and third son of Bezalel Zaritsky, a Talmudic scholar, and Hannah Tannenbaum; at 7 & 8 he eagerly learned chess, arithmetic, and algebra from his oldest brother, Moses. His older brother, Shepsel, taught him to ice-skate on the Mukhavets, a small tributary of the Bug river. His first studies were in the Ukraine (at Vladimir-Volynski in 1910, moved to Chernigov in 1914, and entered the University of Kiev in 1918); he chose the name “Oscar Zariski” when he was a graduate student in Rome where he studied with Guido Castelnuovo, Federigo Enriques, and Francesco Severi, in the 1920’s—and where he met Yole Cagli, whom he married in the garden of his mother’s old stone house in Kobryń on September 11, 1924, “... on the sixth day after the Sabbath, the thirteenth day of the month of Elul, the year 5684 after creation ... ”. just days before receiving his Rome doctorate. Then came a Rockefeller Fellowship at Rome in 1926; and, with help from Solomon Lefschetz, a postgraduate Johnston Scholarship at Johns Hopkins for 1927–28; winding-up as chair at Harvard in 1958. His students include Abhyankar, Artin, Mumford, and Hironaka. He was about 25 when he published his first paper; almost 50 when he did his great work on holomorphic functions; and completed his work on equisingularity when he was almost 80. Zariski lead the way to rigorizing the foundations of algebraic geometry by means of modern abstract algebra—followed quickly by the two revolutions of André Weil (c. 1945) and Alexander Grothendieck (c. 1960). In 1981 Zariski shared the $100,000 Wolf Prize with Lars Ahlfors. Reference: The Unreal Life of Oscar Zariski, by Carol Parikh, Academic Press, 1991. ISBN 0-12-545030-3. QA29.Z37P37 1990.

Zurkowski, Victor. ⟨victor@utstat.toronto.edu⟩. Toronto, Canada.