Dear friends,

The Real Jacobian Conjecture has been proved to be FALSE by Serguey PINCHUK (May 20, 1994). Below I describe his proof. All the formulas can be easily checked by a computer algebra system like MAPLE (I did that).

**COUNTEREXAMPLE**

Define in \( k[x, y] \) the following polynomials:

\[

t = xy - 1, \\
h = t(xt + 1), \\
f = ((h + 1)/x)(xt + 1)^2, \\
p = f + h, \quad \text{and} \\
Q = -t^2 - 6th(h + 1).
\]

Then one can check that

\[
J(p, Q) = J^+ - fv,
\]

where

\[
J^+ = t^2 + (t + f(13 + 15h))^2 + f^2 \\
\quad \text{and} \\
v = v(f, h) = f + f(13 + 15h)^2 + 12h + 12h^2
\]

**Lemma.** There exist a polynomial \( u = u(f, h) \) such that \( J(p, u) = -fv \).

**Proof:** By the Chain rule \( J(f + h, u) = (-f)(du/dh - du/df) \), where one uses that \( J(f, h) = -f \) (also this last formula is easy to check). So one has to solve

\[
\frac{du}{dh} - \frac{du}{df} = v(f, h)
\]

which is easy, for example

\[
u = 170fh + 91h^2 + 195fh^2 + 69h^3 + 75h^3f + (75/4)h^4.
\]

Now put \( q = Q - u \), then by (*) and Lemma we get \( J(p, q) = J^+ \). Observe that \( J^+ > 0 \) on \( \mathbb{R}^2 \) since it is a sum of squares, which can only be zero if both \( t \) and \( f \) are zero. But if \( t = 0 \) then \( f = 1/x \), so \( f \) cannot be zero.

Finally put \( F = (p, q) \). It remains to see that \( F \) is not a global diffeomorphism. Then Pinchuk finishes his proof as follows: “\( p = 0 \) contains the set \( xt + 1 = 0 \), which can be written in the form \( y = (x - 1)/x^2 \). Thus the set \( xt + 1 = 0 \) is a disconnected algebraic set. This is impossible if \( (p, q) \) is a global diffeomorphism.”

END!
Acknowledgement:
I like to thank Armengol Gasull for sending me a Fax of a preprint of Pinchuk’s paper.
With the most friendly regards, Arno van den Essen.

*Note added December 25, 1995, by G. H. Meisters:*
Pinchuk’s paper was published as follows—


Pinčuk gives a beautiful example of a non-injective polynomial mapping from $\mathbb{R}^2$ into itself, of degree $(p, q) = (10, 25)$, whose Jacobian determinant is everywhere positive on $\mathbb{R}^2$. The more famous Jacobian Conjecture of O.-H. Keller remains open: A Polynomial map of $\mathbb{C}^n$ [or of $\mathbb{R}^n$] into itself, for $n > 1$, whose Jacobian determinant is constant on $\mathbb{C}^n$ [or on $\mathbb{R}^n$, respectively], is necessarily injective (and also surjective with a polynomial inverse). Another name for *The Real Jacobian Conjecture* is *The Strong Jacobian Conjecture*. 