

DISCRETE DYNAMICS EXERCISES

1. Write the model

$$p_{t+1} = ap_t + bq_t + cr_t, \quad q_{t+1} = dp_t, \quad r_{t+1} = fq_t$$

in matrix-vector form.

2. Find the dominant eigenvalue and a corresponding eigenvector for each of the following matrices:

(a) $\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix}$

(b) $\mathbf{M} = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$

(c) $\mathbf{M} = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 0 & 0 \\ 2 & 2 & 0 \end{pmatrix}$

3. Find the asymptotic growth rate and the stable age distribution for the system $\mathbf{x}_{t+1} = \mathbf{L}\mathbf{x}_t$,

where $\mathbf{L} = \begin{pmatrix} 0 & 9 & 12 \\ 1/3 & 0 & 0 \\ 0 & 1/2 & 0 \end{pmatrix}$.

4. Consider the general 2×2 Leslie matrix $\mathbf{L} = \begin{pmatrix} f_1 & f_2 \\ p_1 & 0 \end{pmatrix}$. Determine a condition on the parameters so that the dominant eigenvalue is $\lambda_1 = 1$.