(12 each) 1. Solve each of the following problems:

a. $y''' + y'' = 0$
   \[ y = e^{\lambda t} \]
   \[ y = (c_1 + c_2 t) e^{\lambda t} + (c_3 + c_4 t) e^{-\lambda t} \]
   \[ \lambda = 0 \text{ of multiplicity 2 and some } \lambda = i \text{ of multiplicity 2} \]

b. $y'' + 2y' + 3y = 0$
   \[ \frac{-b \pm \sqrt{b^2 - 4ac}}{2} \]
   \[ y = c_1 e^{-3t} \cos(\sqrt{2}t) + c_2 e^{-3t} \sin(\sqrt{2}t) \]
   \[ -1 \pm \frac{2\sqrt{2}}{2} = -1 \pm 1 \]

\[ -2 \]

c. $y'' + y' - 6y = 0, \quad y(0) = 0, \quad y'(0) = 1$
   \[ y'' + y' - 6y = 0 \]
   \[ (y - 2)(y + 3) \]
   \[ y = c_1 e^{3t} + c_2 e^{-2t} \Rightarrow c_1 + c_2 = 0 \Rightarrow c_1 = -c_2 \]
   \[ y' = 3c_1 e^{3t} - 2c_2 e^{-2t} \Rightarrow 3c_1 - 2c_2 = 1 \]
   \[ -3c_2 - 2c_2 = 1 \Rightarrow c_1 = \frac{1}{5} \]
   \[ -5c_2 = 1 \quad c_2 = -\frac{1}{5} \]

(4) 2. Explain (one sentence) which, if any, of the differential equations of problem 1 could be models of a spring-mass system.

letter b could be because

\[ \checkmark \]

- $B$ is a nonnegative constant
- $B$ is damping force, if $B = 0$ then no damping force
- $k$ is spring constant and is positive.
(8) 3. Determine the largest interval for which the problem
\[
(1 + \sin t)y'' + \cos(3t)y' + \frac{t - 2}{t - 2} y = 0, \quad y(0) = 1, \quad y'(0) = 0
\]
is guaranteed to have a unique solution.

\[p(t) = \frac{\cos(3t)}{(1 + \sin t)} \Rightarrow t \neq \frac{\pi}{2} \quad q(t) = \frac{1}{(t - 2)} \Rightarrow t \neq 2 \text{ or } t = \frac{\pi}{2}
\]

\[\frac{-\pi}{2} < t < 2 \quad \text{because } y(0) = 1 \text{ and } y'(0) = 0
\]

(20) 5. A mass of 2 kg stretches a spring 2.5 m. The mass is pulled down an additional meter and released in a medium for which the damping coefficient is exactly half of that needed for critical damping. Describe the subsequent motion.

\[\beta = \frac{k}{2}
\]
\[\beta = 0.12 \text{ damping force}
\]
\[K = \frac{2kq}{8.20} = 24 \text{ kg ft}
\]

\[
\begin{align*}
\text{mass} &= \frac{2kq}{32 + \beta^2} = \frac{1}{16} \text{ kg-sec}^2/\text{ft} \\
\text{critical damping} &= 1y = 9.7 + 9.7m \\
\beta &= 2K \\
y &= (C_1 + C_2 t)e^{-kt}
\end{align*}
\]

\[
\begin{align*}
2.5 \text{ meters} \times \frac{1 \text{ yd}}{3 \text{ ft}} \times \frac{3 \text{ feet}}{1 \text{ yd}} &= 8.20 \text{ ft}
\end{align*}
\]

\[
\frac{1}{16} y'' + .12 y' + .24 y = 0
\]

\[
y'' + 1.92y' + 3.84y = 0
\]

\[
\mu = \frac{-1.92 \pm \sqrt{3.84 - \frac{1.92^2}{4}}}{2} = -.96 \pm 1.71
\]

\[
\omega_0 = \frac{k}{\sqrt{\mu}}
\]

\[
\begin{align*}
\omega_0^2 &= \frac{k}{\mu} \\
\omega_0^2 &= \frac{.24}{\mu} = \sqrt{3.84} = 1.96
\end{align*}
\]

\[
\omega_0 = 1.96
\]

What is \( y(t) \)?

Is it overdamped or underdamped?

\[u(0) = 8.2 \text{ ft} \quad u'(0) = 0 \text{ because it is released initially} \]