

Name \_\_\_\_\_

PAGES 1 AND 2 ARE THE BASIC PART OF THIS EXAM.

(12 each) 1. Solve each of the following problems:

a.  $y''' + y'' = 0$

~~$y^4 + y^2 = 0$~~   
 $y^2(y^2 + 1)$

$y = e^{\lambda t}$

$\lambda^4 \lambda^2 = 0$

$y = (C_1 + C_2 t) e^{0t} + (C_3 + C_4 t) e^{\pm i t}$

~~$\lambda = 0$  of multiplicity 2 & some  $\lambda = \pm i$  of multiplicity 2~~

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b.  $y'' + 2y' + 3y = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2} \Rightarrow \frac{-2 \pm \sqrt{4 - 4(1)(3)}}{2} \Rightarrow -1 \pm \frac{\sqrt{5}}{2}$

$y = C_1 e^{-t} \cos(\sqrt{2}t) + C_2 e^{-t} \sin(\sqrt{2}t)$

$-1 \pm \frac{2\sqrt{2}}{2} = -1 \pm 1$

c.  $y'' + y' - 6y = 0, \quad y(0) = 0, \quad y'(0) = 1$

$y^2 + y - 6y = 0$   
 $(y - 2)(y + 3)$

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~~$y = C_1 e^{3t} + C_2 e^{-2t}$~~   $\Rightarrow C_1 + C_2 = 0 \Rightarrow C_1 = -C_2$

$y' = 3C_1 e^{3t} - 2C_2 e^{-2t} \Rightarrow 3C_1 - 2C_2 = 1$   $C_1 + C_2 = 0$

$y = \frac{1}{5} e^{3t} - \frac{1}{5} e^{-2t}$

$-3C_2 - 2C_2 = 1 \Rightarrow -5C_2 = 1 \Rightarrow C_2 = -\frac{1}{5}$   
 $C_1 = \frac{1}{5}$

(4) 2. Explain (one sentence) which, if any, of the differential equations of problem 1 could be models of a spring-mass system.

letter b could be because,

✓

B is a nonnegative constant

B is damping force, if B=0 then no damping force

K is spring constant and is positive.

L has half positive.

(8) 3. Determine the largest interval for which the problem

$$(1 + \sin t)y'' + \cos(3t)y' + \frac{1}{t-2}y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

is guaranteed to have a unique solution.

$$p(t) = \frac{\cos(3t)}{(1 + \sin t)} \Rightarrow t \neq -\frac{\pi}{2}$$

$$q(t) = \frac{1}{(t-2)(1 + \sin t)} \Rightarrow t \neq 2 \text{ or } \frac{\pi}{2}$$

$$\boxed{-\frac{\pi}{2} < t < 2}$$

because  $y(0)=1$  and  $y'(0)=0$

(20) 5. A mass of 2 kg stretches a spring 2.5 m. The mass is pulled down an additional meter and released in a medium for which the damping coefficient is exactly half of that needed for critical damping. Describe the subsequent motion.

$$\text{mass} = \frac{2 \text{ kg}}{32 \text{ ft/sec}^2} = \frac{1}{16} \frac{\text{kg} \cdot \text{sec}^2}{\text{ft}}$$

critical damping

$$1 \text{ yd} = .9144 \text{ m}$$

$$B = 2K$$

$$y = (c_1 + c_2 t)e^{-kt}$$

$$2.5 \text{ meters} \times \frac{1 \text{ yd}}{.9144 \text{ m}} \times \frac{3 \text{ feet}}{1 \text{ yd}} = 8.20 \text{ ft}$$

$$B = \frac{k}{2}$$

$B = .12$  damping force

$$K = \frac{2 \text{ kg}}{8.20 \text{ ft}} = .24 \frac{\text{kg}}{\text{ft}}$$

$$\frac{1}{16} y'' + .12 y' + .24 y = 0$$

$$y'' + 1.92 y' + 3.84 y = 0$$

$$u = \frac{-1.92 \pm \sqrt{3.84 - \frac{1.92^2}{4}}}{2} = -.96 \pm 1.71i$$

$$\omega_0^2 = \frac{k}{m}$$

$$\omega_0^2 = \frac{.24}{\frac{1}{16}} = \sqrt{3.84} = 1.96$$

$$\omega_0 = 1.96$$

Period  $T = \frac{2\pi}{\omega_0} = \frac{2\pi}{1.96}$   
 This is not correct.

What is  $y(t)$ ?

Is it overdamped or underdamped?

$u(0) = 8.2 \text{ ft}$   $u'(0) = 0$  because it is released initially

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