

40 N

Name _____

(12 each) 1. Solve each of the following problems:

a. $y^{(5)} + ay'''' + by''' + cy'' + dy' = 0$, where the characteristic polynomial is $(\lambda+1)^2 * (\lambda^3 + 6\lambda^2 + 5\lambda)$
 $(\lambda=0) (\lambda+1) (\lambda+5)$
 $(\lambda=-1) (\lambda=-1) (\lambda=-5)$

-10

$y = c_1 + c_2 e^{-5t}$

What about the triple root $\lambda = -1$?

b. $y'' + 4y' + 5y = 0$
 $y^2 + 4y + 5 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2} \Rightarrow \frac{-4 \pm \sqrt{16 - 4(1)(5)}}{2} \Rightarrow \frac{-4 \pm \sqrt{16 - 20}}{2}$

✓

$y = c_1 e^{-2t} \cos(t) + c_2 e^{-2t} \sin(t)$

-2 ± i

c. $y'' - 6y' + 9y = 0, y(0) = 2, y'(0) = -1$

$y^2 - 6y + 9 = 0$
 $(y-3)(y-3)$

where did this come from? $c_1 = 2 - c_2$

$y = c_1 e^{3t} \cos t + c_2 e^{3t} \sin t \Rightarrow 2 = c_1 + c_2$

-10

$y' = 3c_1 e^{3t} \cos t + c_2 e^{3t} \sin t \Rightarrow -1 = 3c_1 + c_2 \Rightarrow -1 = 3(2 - c_2) + c_2$
 $-1 = 6 - 3c_2 + c_2$
 $-7 = -2c_2$

~~$y = \frac{-3}{2} e^{3t} + \frac{7}{2} e^{3t}$~~

$2 = c_1 + c_2$
 $2 = c_1 + \frac{7}{2}$
 $2 - \frac{7}{2} = c_1$
 $\frac{4}{2} - \frac{7}{2} = c_1 \Rightarrow c_1 = -\frac{3}{2}$

(4) 2. Which equation(s) in question 1 can be a linear oscillator? Explain your answer.

$y'' + \beta y' + k^2 y \Rightarrow \beta = 0$, then there is no damping force,

k^2 is a positive constant,

β is nonnegative constant if damping force.

✓

b is most like a linear oscillator because, β is positive and k^2 is positive.

(10) 3. Determine the largest interval for which the problem

$$t(t-2)y'' + 3ty' + 4y = 2, \quad y(3) = 1, \quad y'(3) = 0$$

is guaranteed to have a unique solution.

$$t \neq 2 \quad t \neq 0$$



$$2 < t < \infty$$

(20) 5. A mass weighing 16 pounds stretches a spring 3 inches. The mass is attached to a viscous damper with a damping constant of 2 lb-sec/ft. If the mass is set in motion from its equilibrium position with a downward velocity of 3 in/sec, find its position at any time t . Determine when the mass first returns to its equilibrium position.

$$\text{mass} = \frac{16 \text{ Lbs}}{32 \text{ ft/sec}^2} = \frac{1}{2} \quad \checkmark \quad \checkmark$$

$$K = \frac{16 \text{ Lbs}}{1/4 \text{ ft}} = 64 \quad \checkmark$$

-10

$$\omega_0^2 = \frac{K}{m} \Rightarrow \omega_0^2 = \frac{64}{1/2} = \omega_0 = \sqrt{128}$$

$$\omega_0 = 11.31$$

$$\gamma = 2 \text{ lb/sec/ft} \quad \checkmark \leftarrow \text{damping constant}$$

$$\frac{1}{2}y'' + 2y' + 64y = 4 \quad \checkmark$$

$$y'' + 4y' + 128y = 0 \quad \checkmark$$

Now you have to solve the DE.

$$t = .03125 \text{ sec}$$