(12 each) 1. Solve each of the following problems:

a. \( y^{(5)} + ay''' + by'' + cy' + dy = 0 \), where the characteristic polynomial is \( (\lambda - 4)(\lambda - 1) \)

\[ y = c_1 + c_2 e^t + c_3 e^{-2t} \]
\[ r = 1, \lambda = 4, \lambda = 0 \]
\[ \text{This is a multiple root.} \]

b. \( y'' + 4y' + 6y = 0 \)

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\[ \frac{-b \pm \sqrt{b^2 - 4ac}}{2} = \frac{-4 \pm \sqrt{16 - 4(1)(6)}}{2} = -2 \pm \sqrt{4} \]
\[ y = c_1 e^{-2t} \cos(\sqrt{2}t) + c_2 e^{-2t} \sin(\sqrt{2}t) \]
\[ -2 \pm i \sqrt{2} \]

This is a complex conjugate root.

(6) 2. Under what circumstances can the equation

\[ y'' + \beta y' + \kappa^2 y = 0 \]

represent a linear oscillator? Explain your answer.

\[ \beta = 0, \text{ because if } \kappa = 0 \text{ then there is no spring constant} \]

\[ y'' \neq 0, \text{ because there is the mass.} \]
(8) 3. Determine the largest interval for which the problem
\[(1 + \sin t)y'' + \cos(3t)y' + \frac{1}{t-2}y = 0, \quad y(0) = 1, \quad y'(0) = 0\]
is guaranteed to have a unique solution.

(20) 5. A mass of 2 kg stretches a spring 2.5 m. The mass is pulled down an additional meter and released in a medium for which the damping coefficient is exactly half of that needed for critical damping. Describe the subsequent motion.

\[\text{mass} = \frac{2k_0}{3c + 4k_0/e^2} \Rightarrow \frac{2k_0}{3c + 4k_0/e^2} = \frac{3}{16} \text{ mass with gravity} \]

\[k = \text{spring constant} \Rightarrow 2k_0 = \cdots \]

\[w_0^2 = k/m \Rightarrow \frac{k}{16} = w_0 \Rightarrow \sqrt{\frac{k}{m}} = w_0 \]

\[\gamma = \frac{1}{2} \]

\[u = my'' + y' + ky \Rightarrow u = \frac{1}{16} y'' + \frac{1}{2} y' + ky \]

\[T = \frac{2\pi}{w_0} = \text{Period} \]