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Math 221, section 2 **Progress Test 3-2** 3/1/99

Name [redacted]

(12 each) 1. Solve each of the following problems:

a. $y^{(5)}+ay''''+by'''+cy''+dy'=0$, where the characteristic polynomial is $(\lambda-1)^2*\lambda*(\lambda^2-5\lambda+4)$

$(\lambda-4)(\lambda-1)$

$\lambda=1, \lambda=4, \lambda=0$

$y = c_1 + c_2e^t + c_3e^{4t}$

-6

This is a multiple root.

b. $y''+4y'+6y=0$

$y^2+4y+6=0$

$\frac{-b\pm\sqrt{b^2-4ac}}{2} \Rightarrow \frac{-4\pm\sqrt{16-4(1)(6)}}{2} \Rightarrow -2\pm\sqrt{16-24}$

$-2\pm i\sqrt{2}$

✓

$y = c_1e^{-2t}\cos\sqrt{2}t + c_2e^{-2t}\sin\sqrt{2}t$

c. $y''+4y'+4y=0, \quad y(0)=1, \quad y'(0)=-1$

$y^2+4y+4=0$

$(y+2)(y+2) \Rightarrow -2$

$y = c_1e^{-2t} + c_2e^{-2t} \Rightarrow 1 = c_1 + c_2$

$y' = -2c_1e^{-2t} - 2c_2e^{-2t} \Rightarrow -1 = -2c_1 - 2c_2$

$-1 = -2(c_1+c_2)$

$\frac{1}{2} = c_1+c_2$

$c_1 = ? \quad c_2 = ?$

-8

(6) 2. Under what circumstances can the equation

$$y''+\beta y'+\kappa^2 y=0$$

represent a linear oscillator? Explain your answer.

~~$\beta=0 \Rightarrow$ damping force.~~

✓ K^2 is not zero, because if $K=0$ then there is no spring constant

~~$y''\neq 0$, because this~~ is the mass.

-2

(8) 3. Determine the largest interval for which the problem

$$(1+\sin t)y'' + \cos(3t)y' + \frac{1}{t-2}y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

is guaranteed to have a unique solution.

$(1+\sin t)y'' = -\cos(3t)y' - \frac{1}{t-2}$ $\qquad$ $\frac{1}{t-2} = -(1+\sin t)y'' - \cos(3t)y'$

-9 $y'' = \frac{-\cos(3t)y' - \frac{1}{t-2}}{(1+\sin t)}$ $\qquad$ $-\infty < t < \frac{3\pi}{2}$

(20) 5. A mass of 2 kg stretches a spring 2.5 m. The mass is pulled down an additional meter and released in a medium for which the damping coefficient is exactly half of that needed for critical damping. Describe the subsequent motion.

mass $= \frac{2\text{kg}}{32\text{ ft/sec}^2} = \frac{1}{16}$ } mass with gravity

K = spring constant $\Rightarrow \frac{2\text{kg}}{\quad} =$

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$\omega_0^2 = \frac{k}{m} \Rightarrow \frac{k}{\frac{1}{16}} \Rightarrow \sqrt{\frac{K}{m}} = \omega_0$

$\gamma = \frac{1}{2}$

$u = my'' + \gamma y' + ky \Rightarrow u = \frac{1}{16}y'' + \frac{1}{2}y' + Ky$

$T = \frac{2\pi}{\omega_0}$ = Period