The University of Nebraska-Lincoln
Integration Skills Exam Question Bank

The **UNLintegexam.qu** question bank[1] contains six topics, each having a core of definitions for shared components, along with one to six questions constructed from the core of definitions. The question is always "Find $\int$ \$function dX" for topics 1, 3, 5, and 6 and "Find $\int_{\$from}^{\$to}$ \$function dX" for topics 2 and 4. The answer is always "\$answer", and a mathml version of the answer is always included as a question comment (simplified in the case of definite integrals). The questions differ in the algorithm that produces \$function and \$answer. These algorithms are built from elementary components, making it a simple matter for the questions to be modified by another user.

Note About Using UNLintegexam.qu

Set the feedback so that the "correct answer" is never shown and the "question comment" is always shown. The official "correct answer" is displayed in linear format with so many parentheses that it is essentially unreadable. The "question comment" displays the answer in mathml.

Note About the Exam Design

The design is based on the assumption that students know the derivatives of the functions

$$x^r, \qquad \sin ax, \qquad \cos ax, \qquad \ln x, \qquad e^{ax}, \qquad \arctan x, \qquad \arcsin x.$$

Topic 1 consists of indefinite integrals of linear combinations of several of these elementary functions. Topic 2 consists of definite integrals with integrands that can be simplified to power functions. Topics 3 and 4 consist of indefinite and definite integrals by substitution. Topic 5 consists of integration by parts and topic 6 consists of integrations by partial fractions or by completing the square. The exam could be modified by changing the list of elementary functions, changing the possible combinations of the elementary functions for substitution or parts, or removing some of the parameters.

Notes About the Question Descriptions

Each of the topic descriptions that follow includes (1) specification of the parameters used for questions in the topic and (2) specification of the questions. All questions employ a random choice of the independent variable; in the specifications that follow, X always indicates an independent variable selected randomly from among t, u, v, w, x, y, z. Randomized quantities are specified either as a list separated by commas or a range separated by two periods, as described in the following examples:

- $a, d = 2..9$ means that a and d are (independent unless otherwise noted) randomly chosen integers in the interval $[2, 9]$.

[1]The development of this question bank was funded by NSF grant DUE-0127777.

- $r = \frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$ means that r is an element chosen randomly from the indicated list.

Additional information that does not fit this standard format is given in square brackets.

Topic 1 — elementary indefinite integrals

$c = 1..9; \quad n = 1..4; \quad m = 2..5; \quad k = 4..9; \quad p, a = 0..9; \quad t = -9..9;$
$k - m > 1; \quad pa + (p + a)t^2 > 0$

$P = pX^m(X^n + X^{-k})$, $pX^{(2n+1)/2}$, $\frac{p}{X^m}$, $\frac{p}{\sqrt{X}}$, $p\sqrt{X}$ [probabilities .25, .25, .25, .125, .125]
$A = \frac{a}{1 + X^2}$, $\frac{a}{\sqrt{1 - X^2}}$; $\quad T = \sin cX$, $\cos cX$, e^{cX}

1. \$function = $P + A + T$.

[The order of terms is scrambled.]

Topic 2 — elementary definite integrals

$n = 2..4; \quad c, d = 1..3; \quad C, D = 1..9; \quad C \neq n$

1. \$function = $\frac{cX^{n+1} \pm d}{X^2}$; \$from=1..3; \$to=2..(7-n).
2. \$function = $\frac{cX^n + d\sqrt{X}}{X}$; \$from=1..3; \$to=2..(7-n).
3. \$function = $\frac{CX^n \pm D}{X}$; \$from=1,2; \$to=e,3.
4. \$function = $c + d\sqrt{X}, cX + d\sqrt{X}$; (\$from,\$to) = (1,4), (1,9), (4,9).

[In questions 1–3, the terms are distributed, instead of shown as here, 25% of the time.]

Topic 3 — indefinite substitution

$a, b = 1..6; \quad c = 2..6; \quad n, m = 2..5; \quad k = -2..2; \quad a + b > 2; \quad k \neq 0$

1. \$function = $X^{n-1}e^{bX^n}$; $\quad X^{n-1}e^{-bX^n}$; $\quad (X + a)^{n-1}e^{(X+a)^n}$; $\quad \frac{e^{b\sqrt{X}}}{\sqrt{X}}$;
   $\sin aXe^{b\cos aX}$; $\quad \cos aXe^{b\sin aX}$.
2. \$function = $X^{n-1}\cos cX^n$; $\quad X^{n+1}\cos(a + bX^n)$; $\quad X^{n+1}\cos(bX^n - a)$;
   $X^{n+1}\cos(a - bX^n)$; $\quad e^X\cos(ce^X)$; $\quad e^X\cos(a + be^X)$; $\quad \frac{\cos(c\ln X)}{X}$; $\quad \frac{\cos(c\sqrt{X})}{\sqrt{X}}$.

3. $\$function = X^{n-1}\sin cX^n;\quad X^{n+1}\sin(a+bX^n);\quad X^{n+1}\sin(bX^n-a);$
$X^{n+1}\sin(a-bX^n);\quad e^X\sin(ce^X);\quad e^X\sin(a+be^X);\quad \dfrac{\sin(c\ln X)}{X};\quad \dfrac{\sin(c\sqrt{X})}{\sqrt{X}}.$

4. $\$function = X^{n-1}(a+bX^n)^m;\quad X^{n-1}(bX^n-a)^m;\quad X^{n-1}(a-bX^n)^m;\quad e^{kX}(a+be^{kX})^m;$
$e^{kX}(be^{kX}-a)^m;\quad e^X(a-be^X)^m;\quad \cos cX(\sin cX)^m;\quad \sin cX(\cos cX)^m;$
$\cos X(a+\sin X)^m;\quad \cos X(a-\sin X)^m;\quad \sin X(a-\cos X)^m;\quad \sin X(a+\cos X)^m;$
$\dfrac{(\ln X)^m}{X};\quad \dfrac{(a+\ln X)^m}{X};\quad \dfrac{(a-\ln X)^m}{X}.$

5. $\$function = \dfrac{X^{n-1}}{\sqrt{a+bX^n}};\quad \dfrac{X^{n-1}}{\sqrt{bX^n-a}};\quad \dfrac{X^{n-1}}{\sqrt{a-bX^n}};\quad \dfrac{e^{kX}}{\sqrt{a+be^{kX}}};\quad \dfrac{e^{kX}}{\sqrt{be^{kX}-a}};$
$\dfrac{e^X}{\sqrt{a-be^X}};\quad \dfrac{\cos cX}{\sqrt{\sin cX}};\quad \dfrac{\sin cX}{\sqrt{\cos cX}};\quad \dfrac{\cos X}{\sqrt{a+\sin X}};\quad \dfrac{\cos X}{\sqrt{a-\sin X}};\quad \dfrac{\sin X}{\sqrt{a-\cos X}};$
$\dfrac{\sin X}{\sqrt{a+\cos X}};\quad \dfrac{1}{X\sqrt{\ln X}};\quad \dfrac{1}{X\sqrt{a+\ln X}};\quad \dfrac{1}{X\sqrt{a-\ln X}}.$

6. $\$function = \dfrac{X^{n-1}}{(a+bX^n)^2};\quad \dfrac{X^{n-1}}{(bX^n-a)^2};\quad \dfrac{X^{n-1}}{(a-bX^n)^2};\quad \dfrac{e^{kX}}{(a+be^{kX)^2}};\quad \dfrac{e^{kX}}{(be^{kX}-a)^2};$
$\dfrac{e^X}{(a-be^X)^2};\quad \dfrac{\cos cX}{(\sin cX)^2};\quad \dfrac{\sin cX}{(\cos cX)^2};\quad \dfrac{\cos X}{(a+\sin X)^2};\quad \dfrac{\cos X}{(a-\sin X)^2};\quad \dfrac{\sin X}{(a-\cos X)^2};$
$\dfrac{\sin X}{(a+\cos X)^2};\quad \dfrac{1}{X(\ln X)^2};\quad \dfrac{1}{X(a+\ln X)^2};\quad \dfrac{1}{X(a-\ln X)^2}.$

Topic 4 — definite substitution

$A = 1..2;\quad B = 1..3;\quad b = 1..5;\quad a, c, n = 2..5;\quad m = 4..6;\quad d = 2, 3, 4, 6;\quad A + B > 2$

1. $\$function = (A+BX)^{n-1}, (BX-A)^{m-1}, (A-BX)^{m-1}$; \$from=0,1; \$to=\$from+1..3.
2. $\$function = \dfrac{X^{n-1}}{a+bX^n}, \dfrac{X^{n-1}}{bX^n-a}, \dfrac{X^{n-1}}{a-bX^n}$; \$from=0,1; \$to=2..4; \$to+ $n \le 6$.
3. $\$function = \dfrac{e^X}{a+be^X}, \dfrac{e^X}{be^X-a}, \dfrac{e^X}{a-be^X}$; \$from=0; \$to=1..3.
4. $\$function = \tan cX, \dfrac{\cos X}{a+\sin X}, \dfrac{\cos X}{a-\sin X}, \dfrac{\sin X}{a-\cos X}, \dfrac{\sin X}{a+\cos X}$; \$from=0; \$to=$\frac{\pi}{D}$,
where $D = cd$ and $d > 2$ for the first item and $D = d$ for the others.
5. $\$function = \dfrac{X^{n-1}}{1+X^{2n}}, \dfrac{1}{1+a^2X^2}$; \$from=0,1; \$to=\$from+1..3.

Topic 5 — integration by parts

$a, b = 1..9; \quad c, d, k, p = -9..9; \quad n = 1..6; \quad q = -2, -3, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2};$
$a+b > 2; \quad (a-1)cd = 0; \quad cd+(a-1)(c+d) \neq 0; \quad pk(k-1)(kd-b) \neq 0; \quad \max(k, kd-b) > 0$

1. \$function $= (bX + d)\cos(aX + c)$.
2. \$function $= (bX + d)\sin(aX + c)$.
3. \$function $= (bX + d)e^{kX}$.
4. \$function $= (bX^n + p)\ln X$, $bX^q \ln X$.
5. \$function $= \pm b \arctan aX$, $\pm b \arcsin aX$.

Topic 6 — quadratic denominators

$a = -6..1; \quad b = a + 1..5; \quad c = -4..4; \quad d = -9..9; \quad c(d + 2ac)(d + 2bc)(d + ac + bc) \neq 0;$
$A = 1..5; \quad B = -4..4; \quad C = 1..9; \quad D = -9..9; \quad B(D + 2BC) \neq 0$

1. \$function $= \dfrac{2cX + d}{X^2 - (a + b)X + ab}$.
2. \$function $= \dfrac{2CX + D}{X^2 - 2BX + (A^2 + B^2)}$.