The University of Nebraska-Lincoln
Differentiation Skills Exam Question Bank

(August 2006)

The **UNLdiffexam_August2006.qu** question bank[1] contains ten topics, each having a core of definitions for shared components, along with one to five questions constructed from the core of definitions. The question is always "Find the derivative of $function" for topics 1 through 9 and "Find $\frac{dy}{dx}$ for $function = $R" for topic 10. The answer is always "$answer", and a mathml version of the answer is always included as a question comment. The questions differ in the algorithm that produces $function and $answer. These algorithms are built from elementary components, making it a simple matter for the questions to be modified by another user.

Note About Using UNLdiffexam_August2006.qu

Set the feedback so that the "correct answer" is never shown and the "question comment" is always shown. The official "correct answer" is displayed in linear format with so many parentheses that it is essentially unreadable. The "question comment" displays the answer in mathml.

Note About the Exam Design

The design is based on the assumption that students should memorize the derivatives of the seven functions

$$x^r, \qquad \sin ax, \qquad \cos ax, \qquad \tan ax, \qquad \ln x, \qquad e^{ax}, \qquad n^x.$$

(There is also one question that uses $\arcsin x$ and $\arctan x$; this question can be used or not, depending on the course syllabus.) Topic 1 consists of functions that are linear combinations of several of these elementary functions. Topics 2 through 8 consist of structured problems in which the elementary functions are combined into products, quotients, and compositions as indicated by the topic name. Topic 10 consists of implicit differentiation problems. As a general rule, the individual components get simpler as the structure gets more complicated; thus, the tangent function and n^x appear only in the simpler topics and the implicit differentiation questions do not use the most complicated structures. The exam could be modified by changing the list of elementary functions or by adjusting the difficulty of the basic components of the structured topics. In particular, it would be a simple matter to edit out the tangent and/or n^x functions. It would also be simple to insert other functions, such as arcsine and arctangent, into topic 1; additional functions could be inserted into the structured questions also, but it would require some expertise in writing EDU algorithms and using the built-in ${mathml} function.

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Notes About the Question Descriptions

Each of the topic descriptions that follow includes (1) specification of the parameters used for questions in the topic, (2) specification of the component functions used in questions in the topic, and (3) specification of the questions constructed from the components. Many of the questions employ a random choice of the independent variable; in the specifications that follow, X always indicates an independent variable selected randomly from among t, u, v, w, x, y, z. Randomized quantities are specified either as a list separated by commas or a range separated by two periods, as described in the following examples:

- $a, d = 2..9$ means that a and d are (independent unless otherwise noted) randomly chosen integers in the interval $[2, 9]$.
- $r = \frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$ means that r is an element chosen randomly from the indicated list.

Additional information that does not fit this standard format is given in square brackets.

Topic 1 — elementary functions

$a, j, n = 2..9$; $\quad c, h = 0..5$; $\quad d = 1..6$; $\quad k = -6..6$; $\quad r = \frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$;
$b = -8, -6, -4, \ldots, 8$; $\quad bk(k-1) \neq 0$

$U = de^{kX}$, $d \ln(nX)$, n^X; $\quad P = bX^r$, bX^{-j}, $b\sqrt{X}$
$S = c \sin aX$, $c \cos aX$, $c \tan aX$; $\quad T = c \arcsin X$, $c \arctan aX$

1. Find the derivative of $S + U + P + h$.
2. Find the derivative of $T + U + P + h$.

[The order of terms is scrambled in both questions. Only one of these should be used.]

Topic 2 — products

$a, c, j, n = 2..8$; $\quad b = -9..9$; $\quad k = -6..6$; $\quad bk(k-1) \neq 0$

$P = aX^n + b$, $aX^n + bX$, $aX^2 + bX + c$; $\quad Q = a\sqrt{X} + b$, $aX^{-j} + b$, $aX^{-j} + bX$
$S = \sin aX$, $\cos aX$, $\tan aX$; $\quad T = e^{-cX}$, $e^{-cX} + b$, $e^{kX} + bX$
[independent values of a, b, c, n are generated for each of P, Q, S, T]

1. Find the derivative of PQ, QP, PT, QT.
2. Find the derivative of PS, QS, TS.

Topic 3 — quotients

$a = 2..9; \quad b = -9..9; \quad c, n = 2..6; \quad k = -6..6; \quad bk(k-1) \neq 0$

$P, Q = aX^n + b,\ aX^n + bX,\ aX^2 + bX + c; \quad T = ce^{kX} + b,\ ce^{kX} + bX;$
$S = \sin aX,\ \cos aX,\ \tan aX$
[independent values of a, b, c, n are generated for each of P, Q, S, T]

1. Find the derivative of $\frac{P}{Q}$, $\frac{P}{T}$, $\frac{T}{Q}$.
2. Find the derivative of $\frac{P}{S}$, $\frac{Q}{S}$, $\frac{T}{S}$, $\frac{S}{P}$, $\frac{S}{Q}$, $\frac{S}{T}$.

Topic 4 — compositions

$a = 2..9; \quad b = -9..9; \quad c, n = 2..6; \quad k = -6..6; \quad bk(k-1) \neq 0$

$P = X^n + b,\ X^n + bX; \quad Q = aX^n + b,\ aX^n + bX,\ \sqrt{X} + b; \quad S = \sin aX,\ \cos aX;$
$T = e^{-cX} + b,\ e^{kX} + bX; \quad U = ae^{kX} + b,\ a \ln X,\ n^X$

1. Find the derivative of $\sqrt{P}$, $\sqrt{S}$, $\sqrt{T}$.
2. Find the derivative of S^n, T^n.
3. Find the derivative of e^Q, e^S.
4. Find the derivative of $\ln Q$, $\ln S$.
5. Find the derivative of $\sin Q$, $\cos Q$, $\sin U$, $\cos U$.

Topic 5 — quotients with an embedded product

$a = 2..9; \quad b, B = -9..9; \quad c, n, N = 2..6; \quad k = -6..6; \quad bBk(k-1) \neq 0$

$P = X^n + b,\ X^n + bX; \quad Q = X^N; \quad R = \sin aX,\ \cos aX; \quad S = \sin X,\ \cos X;$
$T = e^{-cX} + B,\ e^{kX} + BX; \quad U = e^{-cX},\ \ln X$

1. Find the derivative of $\frac{QU}{P}$, $\frac{QS}{P}$, $\frac{US}{P}$.
2. Find the derivative of $\frac{QU}{R}$, $\frac{QS}{T}$, $\frac{Q}{US}$.

Topic 6 — products with a composite factor

$a = 2..9; \quad b = -9..9; \quad c, v, n, m = 2..6; \quad d = -2..2; \quad k = -6..6; \quad bk(k-1) \neq 0$

$P = x^n + b,\ x^n + bx; \quad S = \sin x,\ \cos x; \quad T = e^{-cx} + b,\ e^{kx} + bx; \quad U = e^{kx},\ a \ln x;$
$F = x^m,\ x^m + d,\ e^{-vx},\ \sin vx,\ \cos vx$

1. Find the derivative of FP^{-c}, $F\sqrt{P}$.
2. Find the derivative of FS^n, FT^n.
3. Find the derivative of Fe^{x^n}, Fe^S. $[F \neq e^{-vx}]$
4. Find the derivative of $F \ln P$.
5. Find the derivative of $F \sin x^n$, $F \cos x^n$, $F \sin U$, $F \cos U$. $[F \neq \sin vx,\ \cos vx]$

Topic 7 — compositions of products

$a = 2..9; \quad b = -9..9; \quad n, m = 2..6; \quad p = 3..9; \quad k = -6..6; \quad bk(k-1)(m-n) \neq 0$

$F = X^n \sin X,\ X^n \cos X,\ X^n \ln X,\ X^n e^{-aX},\ e^X \sin X,\ e^X \cos X,\ e^X \ln X$
$H = bX^m,\ b \cos X,\ b \sin X,\ b \ln X,\ be^{kX},\ b\pi^m$

1. Find the derivative of $(F + H)^p$.
2. Find the derivative of $\ln(F + H)$. $[F \neq X^n \ln X,\ e^X \ln X; \quad H \neq b \ln X]$
3. Find the derivative of $\sin(X^n \ln X)$, $\cos(X^n \ln X)$, $\sin(X^n e^{aX})$, $\cos(X^n e^{aX})$.

Topic 8 — compositions of compositions

$a = 1..9; \quad b = -9..9; \quad n = 2..6; \quad m = 3..8; \quad k = -6..6; \quad bk(k-1)(m-n) \neq 0$

$P = X^n + b,\ X^n + bX,\ X^2 + bX + a; \quad S = \sin X,\ \cos X; \quad T = e^{-cX} + b,\ e^{kX} + bX$

1. Find the derivative of $(\ln P)^m$, $(\ln S)^m$, $(\sin P)^m$, $(\cos P)^m$, $(\sin T)^m$, $(\cos T)^m$.
2. Find the derivative of e^{S^n}, $e^{\sin P}$, $e^{\cos P}$.
3. Find the derivative of $\ln(\sin P)$, $\ln(\cos P)$, $\ln(\sin T)$, $\ln(\cos T)$.
4. Find the derivative of $\sin T^n$, $\cos T^n$, $\sin e^P$, $\cos e^P$, $\sin(\ln P)$, $\cos(\ln P)$.

Topic 9 — quotients with an embedded composition

$a, A = 2..9; \quad b, B = -9..9; \quad c, n, N = 2..6; \quad k = -6..6; \quad bBk(k-1) \neq 0$

$P = aX^n + b,\ aX^n + bX; \quad Q = AX^N; \quad R = \sin aX,\ \cos aX; \quad S = \sin X,\ \cos X;$
$T = e^{-cX} + B,\ e^{kX} + BX; \quad U = e^{-cX},\ \ln X$

1. Find the derivative of $\frac{S^N}{P}$, $\frac{\ln(Q+B)}{P}$, $\frac{\ln S}{P}$, $\frac{\sin Q}{P}$, $\frac{\cos Q}{P}$.
2. Find the derivative of $\frac{T^N}{P}$, $\frac{e^Q}{P}$, $\frac{e^S}{P}$, $\frac{\sin U}{P}$, $\frac{\cos U}{P}$.
3. Find the derivative of $\frac{S^N}{T}$, $\frac{\ln(Q+B)}{T}$, $\frac{\ln S}{T}$, $\frac{\sin Q}{T}$, $\frac{\cos Q}{T}$.
4. Find the derivative of $\frac{T^N}{R}$, $\frac{e^Q}{R}$, $\frac{\ln(Q+B)}{R}$.
5. Find the derivative of $\frac{P}{\sin Q}$, $\frac{P}{\cos Q}$, $\frac{P}{\ln S}$, $\frac{T}{\sin Q}$, $\frac{T}{\cos Q}$, $\frac{T}{\ln S}$.

Topic 10 — implicit differentiation

$a = 2..9; \quad b = -3..3; \quad c = 1..9; \quad j, m, n, p = 3..6; \quad b \neq 0$

$G = e^y,\ \sin y,\ \cos y; \quad Q = x^p,\ e^x,\ \sin x,\ \cos x; \quad R = x^n,\ e^{ax},\ \sin ax,\ \cos ax; \quad F = \text{“}\sin\text{”},\ \text{“}\cos\text{”}$

1. Find $\frac{dy}{dx}$ for $R + G + x^j y^m = c$.
2. Find $\frac{dy}{dx}$ for $by^m + R + x^j G = c$.
3. Find $\frac{dy}{dx}$ for $F(y^m + x^p) = x^n$, $F(y^m + x^p) = e^{ax}$.