Limits Question Bank

Topic 1 — experiments

1. Compute $\dfrac{1-\cos mx}{x^2}$ for $x = 0.1$, 0.01, and 0.001.

   Use these values to guess the exact value of $\lim\limits_{x\to 0} \dfrac{1-\cos mx}{x^2}$.

   $m = 2..9$

2. Estimate $\lim\limits_{x\to 0} \dfrac{n^x - 1}{x}$. Report your answer to the nearest 0.001.

   $n = 2..8$

Topic 2 — computing limits by factoring

1. (a) Let $f(x) = (x^2 - ax)/(x - a)$. What is f(a)? (multiple choice)

   (b) Compute $\lim\limits_{x\to a} \dfrac{x^2 - ax}{x - a}$.

   $a = 1..9$

2. Compute $\lim\limits_{x\to a} \dfrac{x^2 - (a+b)x + ab}{x^2 - (a+c)x + ac}$.

   $a, b, c = -5..5 \quad abc(a-b)(a-c)(b-c) \neq 0$

Topic 3 — continuity

All of these questions have the same parts:

- Compute $\lim_{x\to a^-} f(x)$.
- Compute $\lim_{x\to a^+} f(x)$.
- What can you say about the continuity of f? (multiple selection)
  - f is continuous at a.
  - f has a removable discontinuity at a.
  - f has a non-removable discontinuity at a.
  - f is continuous at all points other than a.

1. Let
$$f(x) = \begin{cases} mx + b & x \leq a \\ nx + c & x > a \end{cases}.$$

   $a = \pm 1 \quad b = 1..4 \quad m, n = -3, -2, 2, 3 \quad m \neq n \quad c = ma + b - na > 0$

2. Let
$$f(x) = \begin{cases} mx + b & x \leq a \\ \frac{nx+c}{px-d} & x > a \end{cases}.$$

   $a = \pm 1 \quad b, c, d = 1..4 \quad m, n, p = -3, -2, 2, 3 \quad na + c \neq (ma + b)(pa - d) \quad pa \neq d$

3. Let
$$f(x) = \begin{cases} mx+b & x \le a \\ \frac{nx+c}{px-d} & x > a \end{cases}.$$
$a = \pm 1 \quad b, d = 1..4 \quad m, n, p = -3, -2, 2, 3 \quad c = (ma+b)(pa-d) - na > 0 \quad a > d/p$

4. Let
$$f(x) = \begin{cases} mx+b & x \le a \\ \frac{nx+c}{px-d} & x > a \end{cases}.$$
$a = \pm 1 \quad b, d = 1..4 \quad m, n = -3, -2, 2, 3 \quad p = 2, 3 \quad c = (ma+b)(pa-d) - na > 0 \quad a < d/p$

Topic 4 — limits at infinity

1. Compute $\lim\limits_{x\to\infty} \dfrac{ax^p + b}{cx^p + d}$.

   $p = 2..5 \quad a, c = 1..5 \quad b, d = -5..5 \quad d \neq 0$

2. Compute $\lim\limits_{x\to\infty} \dfrac{ax^p + b}{cx^{p+1} + d}$.

   $p = 2..4 \quad a, c = 1..5 \quad b, d = -5..5 \quad d \neq 0$

3. Compute $\lim\limits_{x\to\infty} \dfrac{ax}{\sqrt{c^2x^2 + d}}$.

   $c = 1..5 \quad a, d = -5..5 \quad ad \neq 0$