

Integrals Question Bank

Note that question texts are not always verbatim. In particular, text enclosed in square brackets is an editorial comment about the question rather than part of the question text.

Topic 1 — computing sums

1. Compute $\sum_{i=1}^n ai + b$

$$a = 2..5, \quad b = 2..9, \quad n = 30, 40, 50, 60$$

2. Let $f(x) = x^2 + bx$, set $\Delta x = d$, and let $x_i = i\Delta x$. Determine $x_1, x_2, x_3, x_4, f(x_1), f(x_2), f(x_3), f(x_4)$, and $\sum_{i=1}^4 f(x_i)\Delta x$

$$b = 2..9, \quad d = 0.1, 0.2, 0.4, 0.5$$

Topic 2 — Riemann sums

1. Let $f(x) = 10x^n + c$. List the evaluation points and corresponding function values used for a right-hand Riemann sum with $n = 4$ on the interval $[a, b]$. Compute the Riemann sum.
 $a = 1..3, \quad n = 2..3, \quad c = 2..9, \quad d = 0.2, 0.4, \quad b = a + 4d$

2. Let $f(x) = 10x^n + c$. List the evaluation points and corresponding function values used for a left-hand Riemann sum with $n = 4$ on the interval $[a, b]$. Compute the Riemann sum.
 $a = 1..3, \quad n = 2..3, \quad c = 2..9, \quad d = 0.2, 0.4, \quad b = a + 4d$

3. Let $f(x) = 10x^2 + c$. List the evaluation points and corresponding function values used for a midpoint Riemann sum with $n = 4$ on the interval $[a, b]$. Compute the Riemann sum.
 $a = 1..3, \quad c = 2..9, \quad d = 0.2, 0.4, \quad b = a + 4d$

Topic 3 — exact computation of quadratic integrals

1. Consider the right-hand Riemann sum representing the area under the graph of $3x^2 + 2ax$ from 0 to b using n equal subdivisions.

(a) What is Δx ? (Your answer should be a function of n .)

(b) What is the general formula for x_i in terms of n ?

(c) What is the general formula for $f(x_i)\Delta x$ in terms of n ?

(d) Use the [given] sum formulas to compute the Riemann sum in terms of n .

(e) Compute $\int_0^b 3x^2 + 2ax$ by taking the limit of the Riemann sum as $n \rightarrow \infty$.

$$a = 1..10, \quad b = 2..5$$

Topic 4 — estimating definite integrals from graphs

1. Determine $\int_0^r f(x) dx$ using the graph of f given below.

[The Maple-generated graph is piecewise linear and includes both positive and negative function values.]

Topic 5 — elementary antiderivatives

1. Find an antiderivative of $ax^n + bx^{1/m} + cx^{-p}$.
 $a, b, c = 1..9, \quad m, n, p = 2..5$

2. Find an antiderivative of $ae^{bx} + \frac{c}{x}$.

$$a = 1..9, \quad b, c = -9..9, \quad b(b-1)c \neq 0$$

3. Find an antiderivative of $f(ax) + \sec^2 x$.
 $a = 2..9, \quad f = \text{“sin”}, \text{“cos”}$

4. Find an antiderivative of $\frac{x^{n-1}}{a + bx^n}$.

$$a = 1..8, \quad b = 2..8, \quad n = 2, 4, 6, 8$$

Topic 6 — definite integrals by the fundamental theorem

All answers must be exact.

1. Compute $\int_a^b cx^n dx$.

$$a = 1..2, \quad b = 2..4, \quad b > a, \quad c = 2..9, \quad n = 1, 2, -2, -3$$

2. Compute $\int_0^b ce^{ax} dx$.

$$c = 2..9, \quad a = -9..9, \quad a(a-1) \neq 0$$

3. Compute $\int_a^b \frac{c}{x} dx$.

$$a = 2..5, \quad b = 6..9, \quad c = 2..9$$

4. Compute $\int_a^b \frac{x^{n-1}}{f(x)} dx$.

$$b = 2..4, \quad A, n = 2..5, \quad B = 1..5, \quad b + n \leq 6$$
$$[f(x), a] = [A + Bx^n, 0], [Bx^n - A, 1], [A - Bx^n, 0]$$

Topic 7 — f values from f'

1. Find an antiderivative of cx^n that passes through the point (a, b) .
 $a = 1..4, \quad b = -8..8, \quad c = 1..9, \quad n = 2..5, \quad a + n \leq 6$

2. An object is originally located at $x = a$. Beginning at time zero, the object moves along the x axis with velocity given by the graph below. Determine the location of the object at time r .

[The graph is the same as that of Question 4-1.]

3. An object is originally located at $x = x_0$. Beginning at time zero, the object moves along the x axis with velocity given by $a + bt + ct^2$. Determine the location of the object at time r .

$$a, b, x_0 = -5..5, \quad c = 1..4, \quad r = 2..3, \quad x(a^2 + b^2) \neq 0$$

Topic 8 — average value

1. Determine the average value of f on the interval $[0, r]$.
[The graph is the same as that of Question 4-1.]
2. Find the average value of $x^2 + 2cx$ on the interval $[a, b]$.
 $a, h = 1.4, \quad b = a + h, \quad c = -4.4, \quad c \neq 0$

Topic 9 — derivatives of integrals

1. Find the derivative of $\int_a^x \ln(bt^n + c)f(dt^{-m}) dt$.
 $a = 0.2, \quad b, c, d = 2.9, \quad m, n = 2.6, \quad f = \text{“sin”, “cos”, “tan”}$
2. Find the derivative of $\int_a^x \sqrt{\sec(bv^n)} dv$.
 $a = 0.2, \quad b = 2.9, \quad n = 2.6$