Derivative Concepts and Applications Question Bank

Calculus texts intermingle material on the concept of the derivative, computation of derivatives, and elementary applications, but they do so in an order that is not rigidly prescribed. With the goal of making a text-independent suite of question banks, we have separated the computation of derivatives using derivative rules out from the derivative concept and applications. The individual instructor needs to be careful not to assign the application questions until the students have learned sufficient computation skills to work them. Of course, one could also change the functions to simplify the computations.

Note that question texts are not always verbatim. In particular, text enclosed in square brackets is an editorial comment about the question rather than part of the question text.

Topic 1 — instantaneous velocity

1. A car moves along the x axis with its position given by $x = \sqrt{bt + c}$. Determine the average velocity from time t to each of the times $t + 0.1$, $t + 0.01$, and $t + 0.001$. Use these results to determine the instantaneous velocity at time t.
   $b = 5..8, \qquad c = 2..5, \qquad t = 1..3$

2. Determine the instantaneous velocity at time t for motion given by $x = at^2 + bt + c$.
   $a, t = 2..4, \qquad b = -5..5, \qquad c = 1..5$

Topic 2 — the derivative by the definition

1. Let $f(x) = x^2 + bx$. Determine $f(c + h)$ and the slope of the secant line from $x = c$ to $x = c + h$ as a function of h. Use the definition of the derivative to find $f'(c)$.
   $b = -5..5, \qquad c = -3..3, \qquad bc(b + 2c) \neq 0$

Topic 3 — derivatives and graphs

1. Mark those statements that are true for the function f whose graph is depicted below. [There are five statements about the sign of f', the sign of f'', whether f is increasing or decreasing, and whether f' is increasing or decreasing. Each statement refers to a specific point on the graph.]

Topic 4 — tangent lines and linear approximations

1. Find the equation of the line that is tangent to the graph of $y = ax^2 + bx + d$ at $x = c$.
   $a = 2..4, \qquad b, d = 1..5, \qquad c = -2..2, \qquad 2ac + b \neq 0, 1$

2. Find the best linear approximation for $\dfrac{ax}{k + x}$ at $x = 0$.

   $a = 2..9, \qquad k = 1..9$

3. Find the best linear approximation for $\dfrac{ax}{k + x}$ at $x = c$.

   $a = 2..9, \qquad c = 1..4, \qquad k = 1..9$

4. Find the best linear approximation for $\sqrt{a^2 + x}$ near $x = 0$ and use it to approximate $\sqrt{q}$.
   $a = 3..9, \qquad b = -2..2, \quad q = a^2 + b, \qquad b \neq 0$

Topic 5 — L'Hôpital's rule

Both questions require students to provide the intermediate step(s) as well as the answer.

1. Compute $\lim_{x\to 0} \frac{ax+bx^2}{\sin cx}$.

   $a = -9..9, \qquad b, c = 2..9, \qquad a(a-1) \neq 0$

2. Compute $\lim_{x\to 0} \frac{1-\cos cx}{ax^2}$.

   $a, c = 2..9$

Topic 6 — critical points

1. Find the critical point(s) of $x^p \ln x$.
   $p = -8..8, \qquad p(p-1)(p+1) \neq 0$

2. Find the critical point(s) of $\frac{x+c}{x^2+ax+b}$.

   $a, c = -5..5, \qquad b = 2..5, \qquad ac \neq 0, \qquad c^2 + b > ac$

Topic 7 — global extrema

Both questions have the same text: "Find the value of x that yields the global minimum of $3x^3 + bx^2 + cx$ on the interval $[-1, 1]$." Both require the exact answer and both include a plot of the function. Both have one clearly-visible local minimum and one clearly-visible local maximum.

1. b and c are chosen so that the global minimum occurs at a critical point.

2. b and c are chosen so that the global minimum occurs at an endpoint.

Topic 8 — finding local extrema

In these questions, only the exact answers are correct.

1. Find the local XXX of $c + bx + ax^3$.
   $a, b = -6..6, \qquad c = 1..6, \qquad ab < 0, \qquad$ XXX="maxima","minima"

2. Find the local maxima and local minima of $x^5 + bx^2 + c$.
   $b = -9..9, \qquad c = 1..9, \qquad b \neq 0$

3. Use the graph of f' shown below to find the local maxima and local minima of f.
   $b = -4..0, \qquad c = b + 4..6, \qquad c > 5, \qquad k = \max((5-b)(5-c), (5+b)(5+c)),$
   $a = 6/k, \qquad s = -1, 1; \qquad f' = sa(x-b)(x-c)$