Course Announcement

Math 928–929, Functional Analysis, Fall 2009

Instructor: David Pitts
Time: M-W-F 10:30–11:20, 112 Teacher's College
Text: John Conway, *A course in functional analysis, 2nd ed.*. Secondary sources (not required) are Edwards, *Functional Analysis: theory and applications* and Rudin, *Functional Analysis.*
Prerequisites: Math 921-2 is preferred, but concurrent enrollment in Math 921 should be fine. Some background with basic complex variables (e.g. Cauchy's integral formula) is helpful, but is not essential.

About Functional Analysis: Functional analysis has its roots in solving systems of equations which have infinitely many equations and infinitely many variables. Such equations arise naturally in differential and integral equations. The subject of functional analysis is now vast and growing rapidly. It is a central subject in modern analysis, and its techniques are used throughout analysis. In addition to differential and integral equations, applications can be found in ergodic theory, harmonic analysis, operator theory, representation theory, and the theory of topological groups. More recently, there has been considerable interest in non-commutative geometry and topology.

One could say that in some sense, functional analysis is infinite dimensional linear algebra. The infinite nature of the objects under study means that it is necessary to use approximations to study them, which is where the analysis arises.

About the Course: We will begin with a thorough study of Hilbert spaces, and then move to the study of compact operators on Hilbert space. We will see how to approximate compact operators with finite rank operators. We will prove the spectral theorem for compact operators (this is a theorem about diagonalization) and apply the theory to the solution of a differential equation.

Following a brief discussion of more general operators and Banach spaces, we will discuss the Hahn-Banach Theorem and its uses. This leads to the subject of convexity theory and duality for Banach spaces, which in turn leads to the theory of locally convex topological vector spaces. Locally convex spaces are fundamental to understanding distribution theory, which is used in a variety of contexts, particularly weak solutions to differential equations.

Math 928 finishes with a discussion and some applications of the principle of uniform boundedness.

Math 929 will discuss linear operators on Banach spaces and their spectra (the spectrum of a linear operator in finite dimensions is the set of eigenvalues), and we will then move back to a discussion of operators on Hilbert spaces, which culminates in the very powerful and elegant general spectral theorem for normal operators. Along the way we will develop the analytic functional calculus, which enables one to define functions of an operator A, e.g. $\exp(A)$ or $\sin(A)$.

The course is theoretical but will also have applications, which can be tailored to student interest.