

M901-2008 Assignment 3: Due Monday September 22

Instructions: Do any three of the following problems.

- (1) Let a group G act transitively on a set S . Let $s \in S$ and let N be a subgroup of G_s such that $N \triangleleft G$. Show that $N < \ker(\pi)$, where $\pi : G \rightarrow \text{Perms}(S)$ is the homomorphism corresponding to the group action. Conclude that $\ker(\pi)$ is the largest normal subgroup of G contained in G_s .
- (2) Let $p < q < r$ be primes and let G be a group of order pqr . Show that G is not simple (i.e., has a nontrivial normal subgroup).
- (3) Show that a group G of order 150 is not simple. (This is the key step for Problem 1 on the January 2007 comprehensive exam, which asks that you prove that G is solvable.)
- (4) Show that a group G of order 105 has a normal subgroup of order 35. (This is Problem 3(a) on the June 2006 comprehensive exam.)
- (5) If G and H are nonabelian groups of order 88 all of whose Sylow subgroups are cyclic, prove that G and H are isomorphic. (This is similar to a problem on the January 2008 comprehensive exam.)