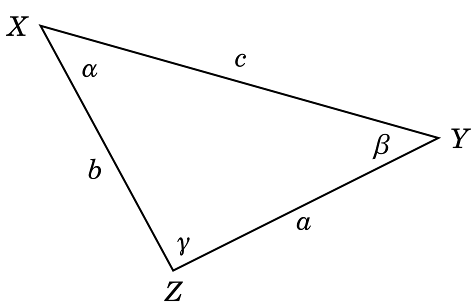
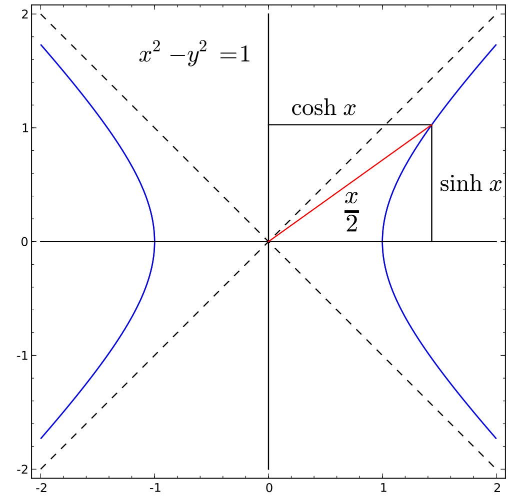
Hyperbolic Trigonometry

Let XYZ be a triangle, with angles α, β, γ and opposite side lengths a, b, c.





	Law of Cosines	Law of Sines	Law of Cosines for Angles
euclidean	$c^2 = a^2 + b^2 - 2ab\cos\gamma$	$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$	AAA not a congruence
spherical*	$\cos c = (\cos a)(\cos b) + (\sin a)(\sin b)\cos\gamma$	$\frac{\sin\alpha}{\sin a} = \frac{\sin\beta}{\sin b} = \frac{\sin\gamma}{\sin c}$	$\cos\gamma = -(\cos\alpha)(\cos\beta) + (\sin\alpha)(\sin\beta)\cos c$
hyperbolic*	$\cosh c = (\cosh a)(\cosh b) - (\sinh a)(\sinh b)\cos\gamma$	$\frac{\sin\alpha}{\sinh a} = \frac{\sin\beta}{\sinh b} = \frac{\sin\gamma}{\sinh c}$	$\cos\gamma = -(\cos\alpha)(\cos\beta) + (\sin\alpha)(\sin\beta)\cosh c$

* For spherical triangles, the side lengths a, b, c are expressed as *central angles*, and hence are normalized by the radius of the sphere. For hyperbolic triangles, the side lengths a, b, c are also normalized by the radius of the hyperbolic plane.

The hyperbolic trigonometric functions are defined as follows:

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{\sinh x}{\cosh x}$$

The hyperbolic sine and the hyperbolic cosine are defined using the "unit hyperbola" $x^2 - y^2 = 1$, where $x/2$ is the area of the region bounded by the x-axis, the line $y = x$, and the hyperbola. This area plays a similar role as the angle does when defining the usual trigonometric functions using the unit circle.