Math 812T Homework Day 4

Problem 1: Pick any two of the points (your choice) constructed for Problem 1 on the previous homework assignment, for example the centroid and the orthocenter. Prove that the only triangles for which the two points are the same point are equilateral triangles.

Problem 2: Suppose that Chef I. Sauce Alees bakes a cake in a triangular pan. She then flips the cake out of the pan and frosts the top (so that the frosted side is the side that was on the bottom when the cake was baked in the pan). To deliver the cake to a Geometry class, she decides to transport the cake in the pan it was baked in. However, what was on the bottom must now be on top, in order to have the frosting on top, but with the bottom on top, the cake is the wrong shape to fit back in the pan! Chef Alees consults a Geometry class, and they tell her that the cake can be cut into three pieces using three straight cuts, such that the resulting pieces can be put into the cake pan with the frosting up. How did the Geometry students explain to make the cuts, for any triangular pan?

Problem 3: Use the law of cosines for sides to find the distance from New York to Los Angeles, given that the Earth is a sphere of radius approximately 4000 miles and that the latitudes for these cities are respectively about $41°$ and $34°$, and the longitudes for these cities are respectively about $74°$ and $118°$. Explain your how you obtain your answer. Compare your result to the actual distance.

Problem 4: The spherical analog of Pythagoras' Theorem for a proper spherical triangle with sides a, b and c, and opposite angles A, B and C respectively, is $\cos(c) = \cos(a)\cos(b)$. It is known (from calculus, for example) that $\cos(x)$ is approximately equal to $1 - x^2/2$ when x is close to 0 (when x is in radians). Use this fact and $\cos(c) = \cos(a)\cos(b)$ to show that $a^2 + b^2 - c^2$ is also close to 0 if a, b and c are close to 0. (This shows that Pythagoras' usual statement is a good approximation for small triangles on a sphere.)