Math 812T Homework Day 3

Due Thursday, June 5.

Problem 1: In GeoGebra, construct a triangle ABC with three movable points. For each item below, add the following lines, and the intersections between the three lines of each item. Put all of the lines in the same figure; it may be helpful to hide some of the lines and points while working on another item. You may use all of the built-in features of GeoGebra (i.e., you do not need to do this with straightedge and compass).

(i) *centroid*: from each vertex, draw a line to the midpoint of the opposite side.
(ii) *circumcenter*: for each side, draw the perpendicular bisector (that is, the perpendicular line through the midpoint of the side).
(iii) *incenter*: from each vertex, draw the angle bisector.
(iv) *orthocenter*: through each vertex, draw an altitude (that is, a line through the vertex that is perpendicular to the opposite side).

Now experiment with your dynamic figure to answer the following questions:

(a) For each item, do the three lines always intersect in a point?
(b) Is the point always inside the triangle?
(c) Do the points always lie on a line? If not, does some subset of 3 of the points always lie on a line? Which subset?

Problem 2: The following problem is claimed to be from a manuscript from about 100 AD in ancient China. *There is a square town of unknown dimensions. There is a gate in the middle of each side. Twenty paces outside the North Gate is a tree. If one leaves the town by the South Gate, walks 14 paces due south, then walks due west for 1775 paces, the tree will just come into view. What are the dimensions of the town?*

Problem 3: The points of the star shown in the figure form a regular pentagon. Assume the circle has unit radius. Let S be the shaded area. Note that 5 copies of the shaded area cover the circle if you allow overlapping. Use this fact with an adjustment for the overlap, to find the area P of the small pentagon at the center of the diagram in terms of S.