Spherical Triangles Worksheet

1) We have heard a few definitions of what a triangle is. On the plane we (more or less) agreed that the following was a good definition: A triangle consists of three non-collinear points (vertices), and three line segments (sides), where the endpoints of the line segments are the vertices. Below are two possible definitions for "triangle" on the sphere. We need to come to a consensus on which we plan to use in our course...(each has its benefits/detriments).

Definition 1: A triangle is the set of three non-collinear points (vertices) which are joined by three straight line segments [great circle segments] (sides), where the endpoints of the line segments are the vertices and it separates the surface into exactly two regions.

Definition 2: A proper triangle consists of three non-collinear points (vertices), and three line segments[great circle segments] (sides), where the endpoints of the segments are the vertices and the length of each segment is less than one half the circumference of the sphere (minor arc).

Discuss in your groups which definition we should use for a spherical triangle in this course.

2) In the plane we have a notion of the interior and exterior of a triangle so that we can talk about the interior angle sum.

a. How is this defined in the plane? How could we define this on the sphere? Is your notion well-defined/specified given the definition of triangle?

3) Using our common definition of triangle:

a. Draw a triangle on the sphere and calculate its angle sum...what do you notice?
b. Can you make a 90-90-90 triangle on the sphere?
c. Does your Venn Diagram characterization of triangles on the plane work for triangles on the sphere? Why or why not? Use the Lenart sphere to make examples.
d. What are the largest and smallest angle sums that are possible on the sphere?
e. Do three points on the equator define a triangle on the sphere? Should it?
f. What is the craziest looking triangle you can make that satisfies our definition?
g. Given three points on a sphere, how many triangles have those points as vertices? Does it depend on the location of the points?

4) What notions do we have from Euclidean Geometry to test if triangles are congruent? Do these notions hold on the sphere?