Given any point P and any line L, how to construct a line through P perpendicular to L:

Steps: (See diagram below.)

1. Draw circle X centered at P intersecting L at two points, D and E.

2. Draw circles Y (centered at D and passing through E) and Z (centered at E and passing through D) meeting at two points C and F.

3. The line through C and F passes through P and is perpendicular to L.

Why is the line CF perpendicular to L? Segments DC and DF have the same length because C and F are on circle Y. Likewise, segments EC and EF have the same length because C and F are on circle Z. But both Y and Z have the same radius, this being the length of segment DE. Thus segments CD, DF, FE and EC all have the same length, so quadrilateral CDEF is a rhombus, but it is a fact that the diagonals of a rhombus (in this case the diagonals are the lines L and CF) are perpendicular. This fact is because the diagonals divide the rhombus up into four congruent triangles, so the four angles where the diagonals cross are congruent, so each must be a right angle.

Why is P on the line CF? Let G be the point where line CF and L meet. Segments PD and PE are the same length since D and E are on circle X. Since triangle DGC and GEC are two of the four congruent triangles mentioned above, we know segments DG and GE have the same length. Thus triangles PGD and PGE are congruent by SSS. Thus angles PGD and PGE are congruent, but together they make a straight angle, so each is a right angle. Thus line GP and line GC both are perpendicular to L at G, so line GP is the same line as line GC which is the same as line CF, so P is on line CF.