Homework, due Tuesday June 3

Problem 1:

Varignon's Theorem: Prove that the midpoints of the sides of any quadrilateral are the vertices of a parallelogram. Hint 1: Consider the diagonals of the quadrilateral and use a triangle similarity property (for example, two triangles are similar if the ratios of the lengths of two sides are the same, as long as the included angle is also the same).

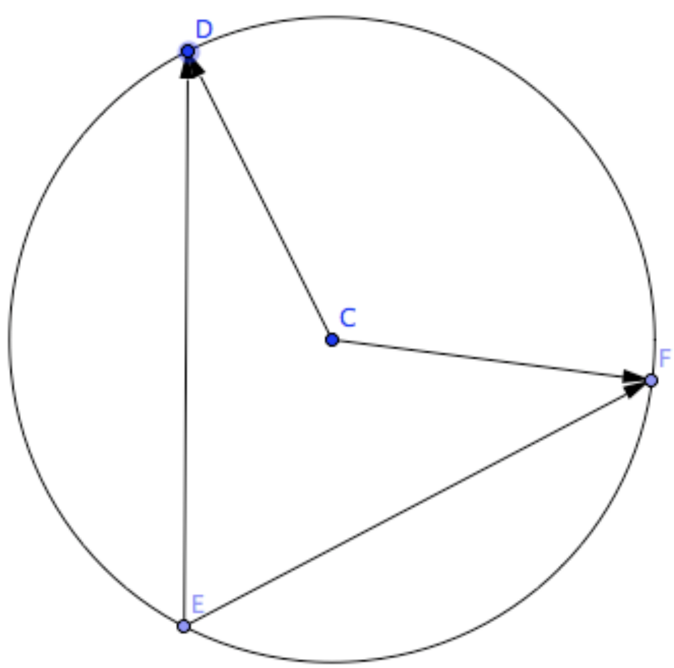
Problem 2:

Look at all triangles two of whose vertices are four inches apart and whose perimeter is 12 inches. You can do this in the following way. Put pins in a piece of cardboard 4 inches apart and make a foot long loop of string around the pins. Use a pencil to pull the loop taut, and keeping it taut, trace out a curve. This curve shows the possible positions for the third vertex of the triangle.

What curve did you get? Can you find a reference on the web to justify your guess? Based on this, what is the largest possible area of one of a triangle with one side of length 4 and perimeter 12? What is the shape of this triangle? Explain your answer.

Problem 3:

Consider a circle X with center C. Let D, E and F be points on X. Let A be the angle through D and F with vertex at E (i.e., angle DEF). Let m(A) be its angle measure. Assume that m(A) is at most 180 degrees. Let B be the angle DCF. Prove that 2m(A)=m(B). (What this shows is, given a point on a circle and a circular arc, the angle with vertex at the point and subtended by the arc is half the angle of the arc.)



Conclude that m(A) = 90 degrees if the line segment from D to F is a diameter of X. Explain how this is related to the 3 squares problem.